Task (error-driven) Learning

• Last time we discussed **self organizing Hebbian learning**
  • Leverage correlations to grow detectors that correspond to things in the world (cats, professors…)

• Today we will discuss **task learning**
  • Task = producing a specific output pattern in response to an input pattern
    • e.g., reading; giving the correct answer to 3 + 3
Task Learning

• Task learning encompasses:

  • Giving an appropriate response to a stimulus
  • Arriving at an accurate interpretation of a situation
  • Generating a correct expectation of what will happen next

• in all of the above cases, there is a correct answer...
Overview

• How well can Hebbian rules support task learning?

• Not well enough! There are some input-output mappings that Hebb can not learn

• Error-correction learning and the delta rule

• Shortcomings of two-layer delta rule networks

• GeneRec: A biologically plausible error-driven learning rule for multilayer networks
Hebbian Task Learning

If you want to learn an input-output association:

- **clamp** the input pattern onto the input layer
- clamp the output pattern onto the output layer
- do Hebbian learning
“Easy” Mapping

- no overlap between inputs
Hebbian learning:
weight \sim P(\text{receiver active} \mid \text{sender active})
Hebbian learning:
weight \sim P(\text{receiver active} \mid \text{sender active})
Hebbian learning:
weight ~ P(receiver active | sender active)
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Hebb can solve the task!
Another (Harder) Mapping

- overlap between inputs
- input units associated with multiple outputs
The mapping is solvable!
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Hebbian learning: weight $\sim P(\text{receiver active} \mid \text{sender active})$
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Can these weights solve the task?

Event_0

Event_1

Event_2

Event_3
Can these weights solve the task?
Event 0 OK!
Can these weights solve the task?
Event 1 OK!
Can these weights solve the task? Event 2 not OK....
Can these weights solve the task? Event 3 not OK....
Weights learned by Hebb =>

<= (one set of)
Weights that solve the task
Solution: Error-Driven Learning

First, we will consider how to do this and later come back to biology and more realistic implementation

- Instead of learning based on correlations, learn based on error: The difference between what the network is supposed to do, and what it actually does

- Error can be indexed using sum squared error (SSE)

\[ SSE = \sum_p \sum_k (t_{kp} - o_{kp})^2 \]

- \( t \) = target output value (what activation is supposed to be) over all output units \( k \), summed across all input patterns \( p \)
- \( o \) = actual output values for each \( k \) unit and input pattern \( p \)
Adjusting Weights to Minimize Error

• Say that we want hidden activity = 1 for this input pattern.

• If you could pick one (of the two) weights to increment, which would you change?
Adjusting Weights to Minimize Error

• Say that we want hidden activity = 1 for this input pattern.

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Adjusting Weights to Minimize Error

- Say that we want hidden activity = 0 for this input pattern.

- If you could pick one (of the two) weights to decrement, which would you change?
Adjusting Weights to Minimize Error

• Say that we want hidden activity = 0 for this input pattern.

• If you could pick one (of the two) weights to decrement, which would you change?
Credit/Blame Assignment

- Error-driven learning is all about figuring out who to blame for mistakes

- If the network makes an error, you should change weights from active input units

- Changing weights from inactive inputs has no effect
The Delta Rule

- The *delta rule* meets the criteria we have outlined for error-driven learning:

\[
\Delta w_{ik} = \epsilon (t_k - o_k) s_i
\]

∀ \( \Delta w_{ik} = \) change in weight

- \( t_k = \) target output value (what activation is supposed to be)
- \( o_k = \) actual output value
- \( s_i = \) input unit activity

- Weight change is proportional to error, and it is also proportional to sending unit activity
Error-driven learning

“hooray for tigers!”

“birds are bad!”

striped  orange  sharp teeth  furry  yellow  chirps
Error-driven learning

“hooray for tigers!”

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The Delta Rule and the “Hard” Problem

• The delta rule can learn the “hard” mapping that thwarted the Hebb rule
What is Target? Activation Phases

a) Minus Phase (expectation)

b) Plus Phase (outcome)

\[ \Delta w_{ik} = \epsilon (o_k^+ - o_k^-) s_i \]
[pat_assoc.proj]
Nature of the Training Signals

a) Explicit Teacher

b) Implicit Expectation

c) Implicit Motor Expectation

d) Implicit Reconstruction
Soft Weight Bounding

Keep weights bounded between 0-1 by exponentially slowing increases, decreases as they approach bounds:

\[
\Delta w_{ik} = [\Delta_{ik}]_+ (1 - w_{ik}) + [\Delta_{ik}]_- w_{ik}
\]

[\Delta_{ik}]_+ = \text{computed weight change if positive (else 0)}.

[\Delta_{ik}]_- = \text{computed weight change if negative (else 0)}.

* reflects biological constraints on number of receptors, etc. (weight can only go so high, low)
“Impossible” Mapping

- Each input unit is linked equally often to each output unit
- Two layer networks using the delta rule can not solve this!
Changing weights to learn Event_0...
Changing weights to learn Event_0...

... hurts performance for Event_2 and Event_3
Add a hidden layer that represents feature **conjunctions** ...
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Add a hidden layer that represents feature conjunctions ...

hidden layer => 1, 3  2, 4  1, 2  3, 4
Add a hidden layer that represents feature conjunctions ...

hidden layer => 1, 3, 2, 4, 1, 2, 3, 4
Add a hidden layer that represents feature **conjunctions** ...
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Error-Driven Learning in Multilayer Networks

- We established that networks with hidden layers can solve problems that two-layer networks can not solve, by re-representing the input patterns.
- How do we train multi-layer networks?
Learning in Multilayer Networks
Learning in Multilayer Networks

target =>
Learning in Multilayer Networks

target =>

Learning in Multilayer Networks
Learning in Multilayer Networks
Learning in Multilayer Networks

target =>

\[
\begin{align*}
\text{target} & \rightarrow \\
& \quad \text{network structure}
\end{align*}
\]
Learning in Multilayer Networks

How do we adjust these connections? =>

target =>
Learning in Multilayer Networks

Intuitively, you want to boost the activity of the middle guys that are well connected to the target unit.

How do we adjust these connections? =>

target =>
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How do we identify units that are well connected to the target unit?

How do we adjust these connections?
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Solution: Propagate activity backwards from the target.
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Intuition: Backward-spreading activity from the target can help us identify pathways to the target (if weights are symmetric).
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Then: change weights to strengthen these pathways.
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Then: change weights to strengthen these pathways.
GeneRec Learning Rule

Compare two conditions:

Minus Phase:
Clamp input
GeneRec Learning Rule

Compare two conditions:

Minus Phase:
Clamp input
GeneRec Learning Rule

Compare two conditions:

Minus Phase:
Clamp input

Plus Phase:
Clamp input and target output
GeneRec Learning Rule

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Clamp input

Plus Phase:
Clamp input and target output
GeneRec Learning Rule

Compare two conditions:

- **Minus Phase:** Clamp input
- **Plus Phase:** Clamp input and target output

For each layer, use the **difference between minus and plus activations** as an error signal and learn using the delta rule.
GeneRec Learning Rule

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For each layer, use the **difference between minus and plus activations** as an error signal and learn using the delta rule.
The goal of error-driven learning is to construct a path from the input to the target output.

**Minus Phase:**
- Clamp input

**Plus Phase:**
- Clamp input and target output
The goal of error-driven learning is to construct a **path** from the input to the target output.

**Minus Phase:**

**Plus Phase:**

The Plus Phase helps identify **bridging units** that are well connected to both the input and the target output, and GeneRec adjusts weights to maximize the activity of these units.
GeneRec: Equations

**Minus Phase:** Clamp input

**Plus Phase:** Clamp input and target output

Basic GeneRec: \[ \Delta w_{ij} = \epsilon (y_j^+ - y_j^-) x_i^- \]
GeneRec: Equations

Basic GeneRec:
\[ \Delta w_{ij} = \epsilon (y_j^+ - y_j^-) x_i^- \]

Two issues: Need weights to be symmetric, and why should we use minus phase sending activity instead of plus phase?
GeneRec: Equations

Basic GeneRec:

Two issues: Need weights to be symmetric, and why should we use minus phase sending activity instead of plus phase?

Solution: Average together plus and minus phase sending activation, and average together feedforward and feedback weight changes.
GeneRec: Equations

Solution: Average together plus and minus phase sending activation, and average together feedforward and feedback weight changes

New and improved GeneRec: (CHL)

\[ \Delta w_{ij} = \epsilon \frac{1}{2} \left[ (x_i^+ + x_i^-)(y_j^+ - y_j^-) + (y_j^+ + y_j^-)(x_i^+ - x_i^-) \right] \]
\[ = \epsilon \left[ (x_i^+ y_j^+) - (x_i^- y_j^-) \right] \]
Remember the “impossible” problem?

It can’t be solved by two-layer networks using the delta rule...
But it can be solved by three layer networks where hidden units represent feature conjunctions....
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Does error-driven learning learn the correct set of weights?
Task Learning: Summary

• Hebbian learning alone is very limited in its ability to learn input-output mappings
  
  • If the input-output mapping happens not to coincide with the correlational structure of the inputs, Hebbian learning fails
  
• Error-driven learning rules (that leverage the difference between what the network was supposed to do, and what it actually did) do better at learning input-output mappings
Task Learning: Summary

- The **delta rule** can learn a wide variety of input-output mappings (including some that Hebb cannot learn) in two-layer networks, but:
  
  - There are some mappings it cannot learn (e.g., the “impossible” mapping)
  
  - It does not apply to networks with more than two layers
Task Learning: Summary

• The GeneRec rule remedies the deficiencies of the simple delta rule

  • It applies to networks with hidden layers
  
  • It can solve tasks that can not be solved by the simple delta rule; this is accomplished by re-representing input patterns...

  • The rule is biologically plausible! Key prerequisites: Bidirectional connectivity, (approximate) symmetry, two “phases” (expectation and outcome)

• Next lecture: Synergies between Error and Hebb => Error + Hebb leads to better learning than Error alone!