Task (error-driven) Learning

- Last time we discussed **self organizing Hebbian learning**
  - Leverage correlations to grow detectors that correspond to things in the world (cats, professors…)
- Today we will discuss **task learning**
  - Task = producing a specific output pattern in response to an input pattern
  - e.g., reading; giving the correct answer to 3 + 3

Task Learning

- Task learning encompasses:
  - Giving an appropriate **response** to a **stimulus**
  - Arriving at an accurate **interpretation** of a **situation**
  - Generating a correct **expectation** of what will happen next
- in all of the above cases, there is a **correct answer**...

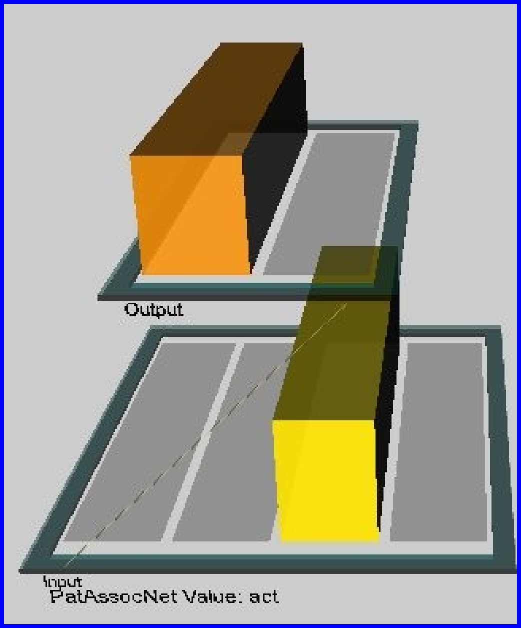
Overview

- How well can Hebbian rules support task learning?
- Not well enough! There are some input-output mappings that Hebb can not learn
- Error-correction learning and the delta rule
- Shortcomings of two-layer delta rule networks
- GeneRec: A biologically plausible error-driven learning rule for multilayer networks

Hebbian Task Learning

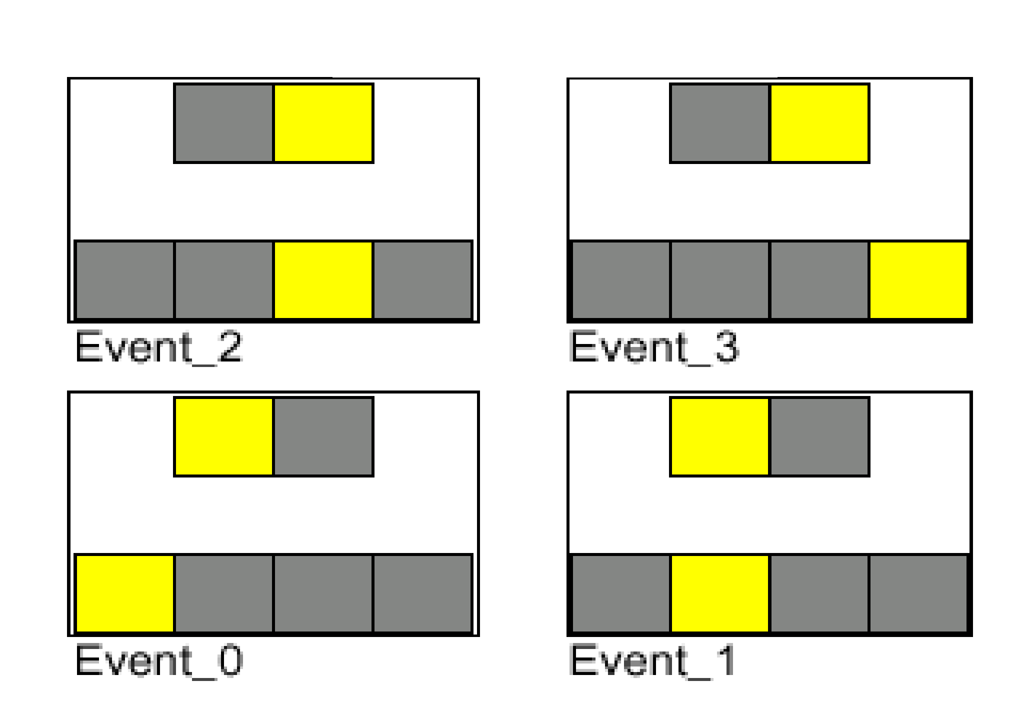
- If you want to learn an input-output association:

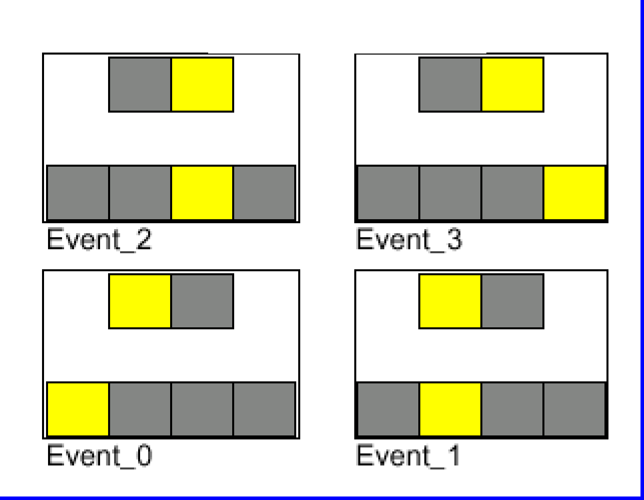
  - **clamp** the input pattern onto the input layer
  - clamp the output pattern onto the output layer
  - do Hebbian learning



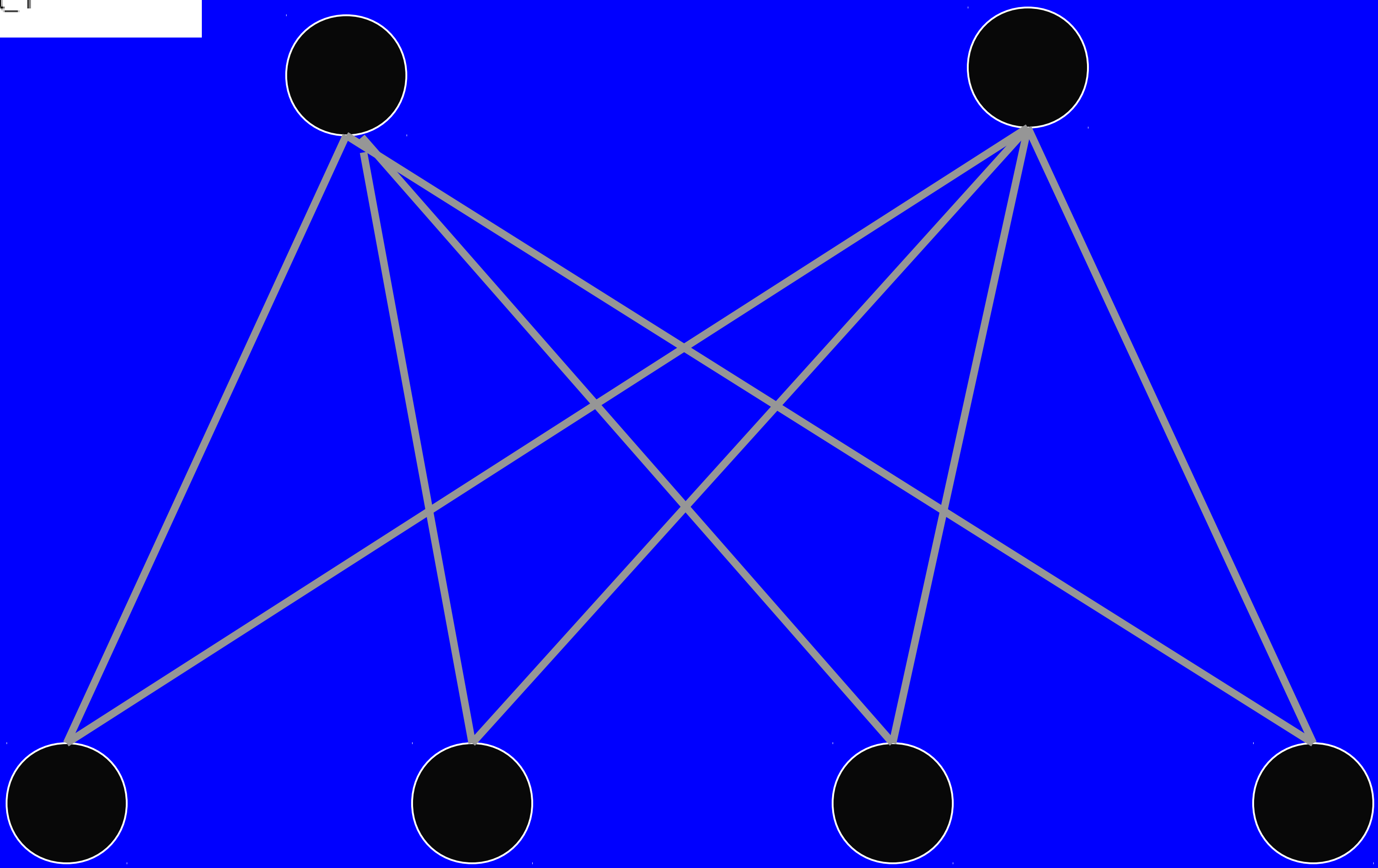
“Easy” Mapping

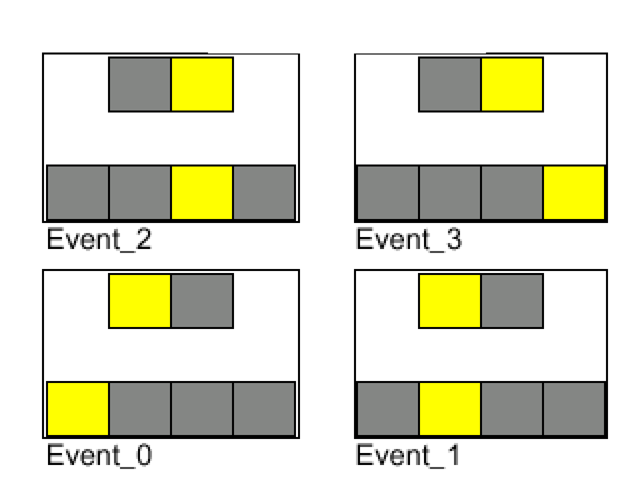
- no overlap between inputs





Hebbian learning:
weight ~ P(receiver active | sender active)





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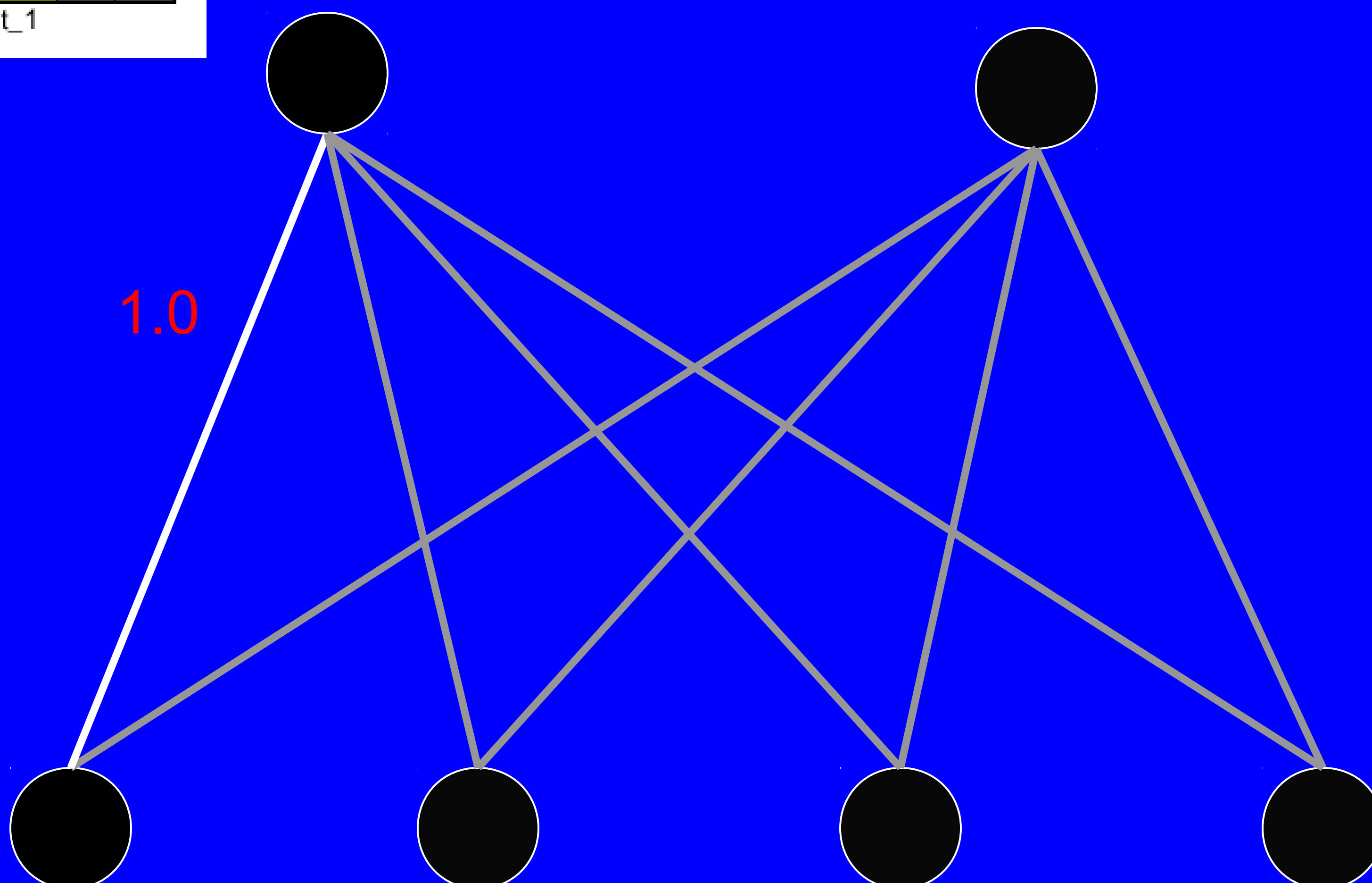

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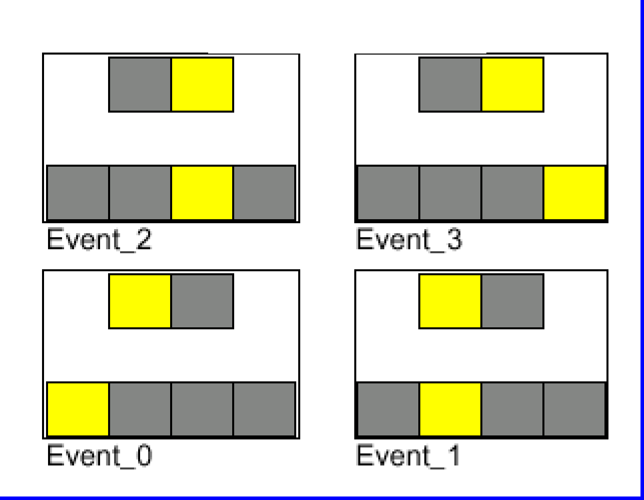

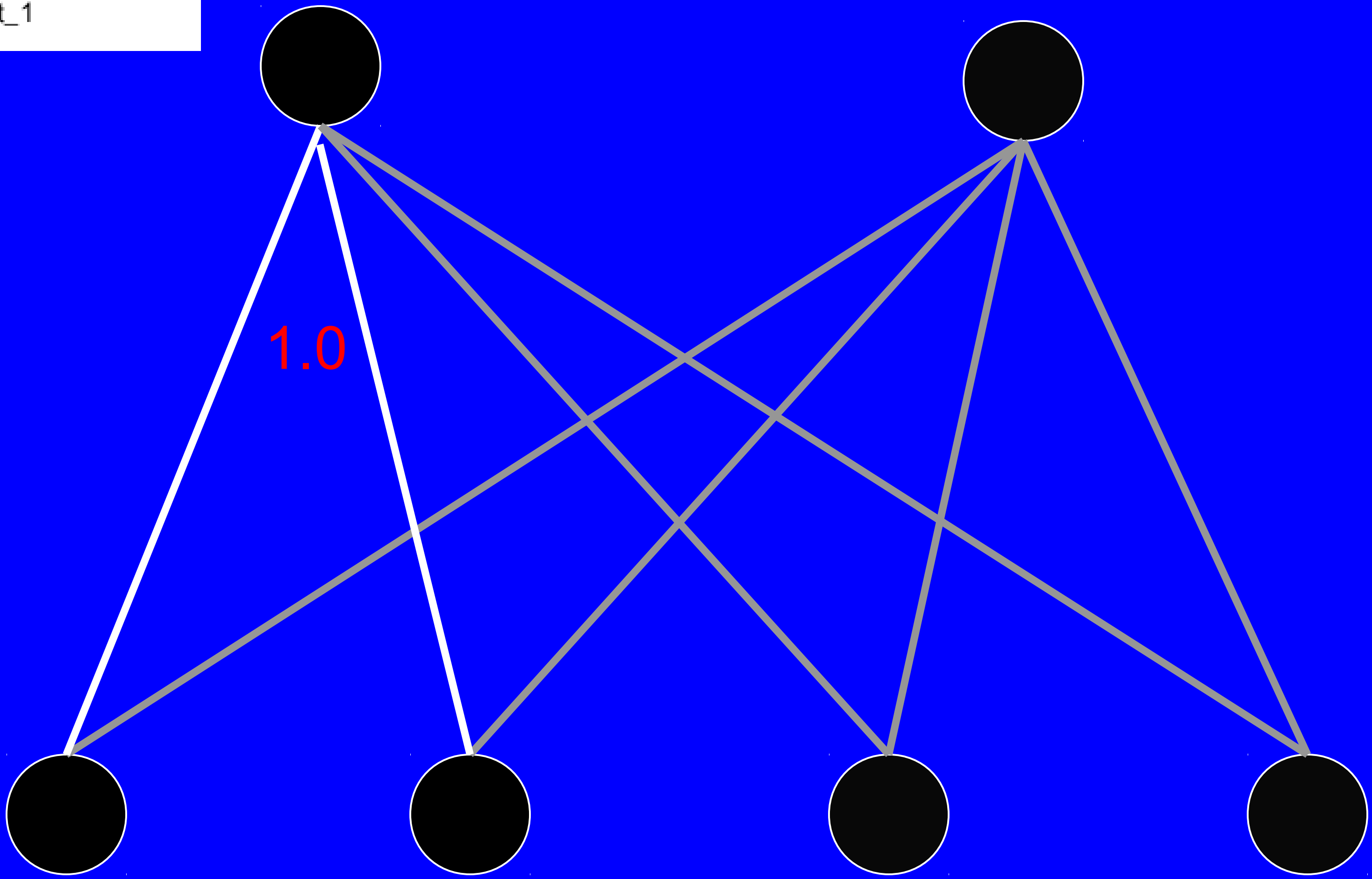

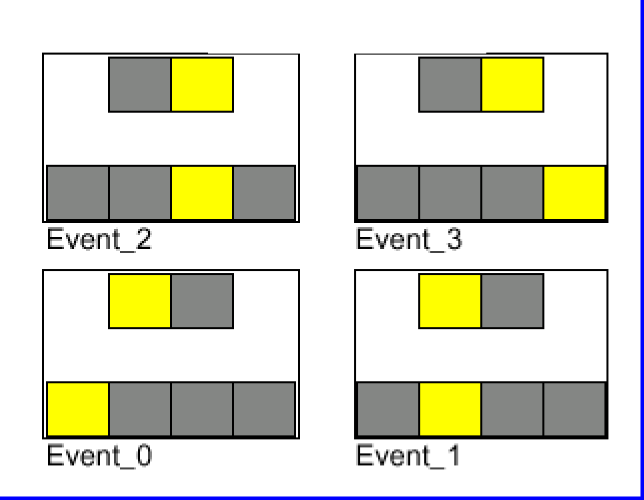

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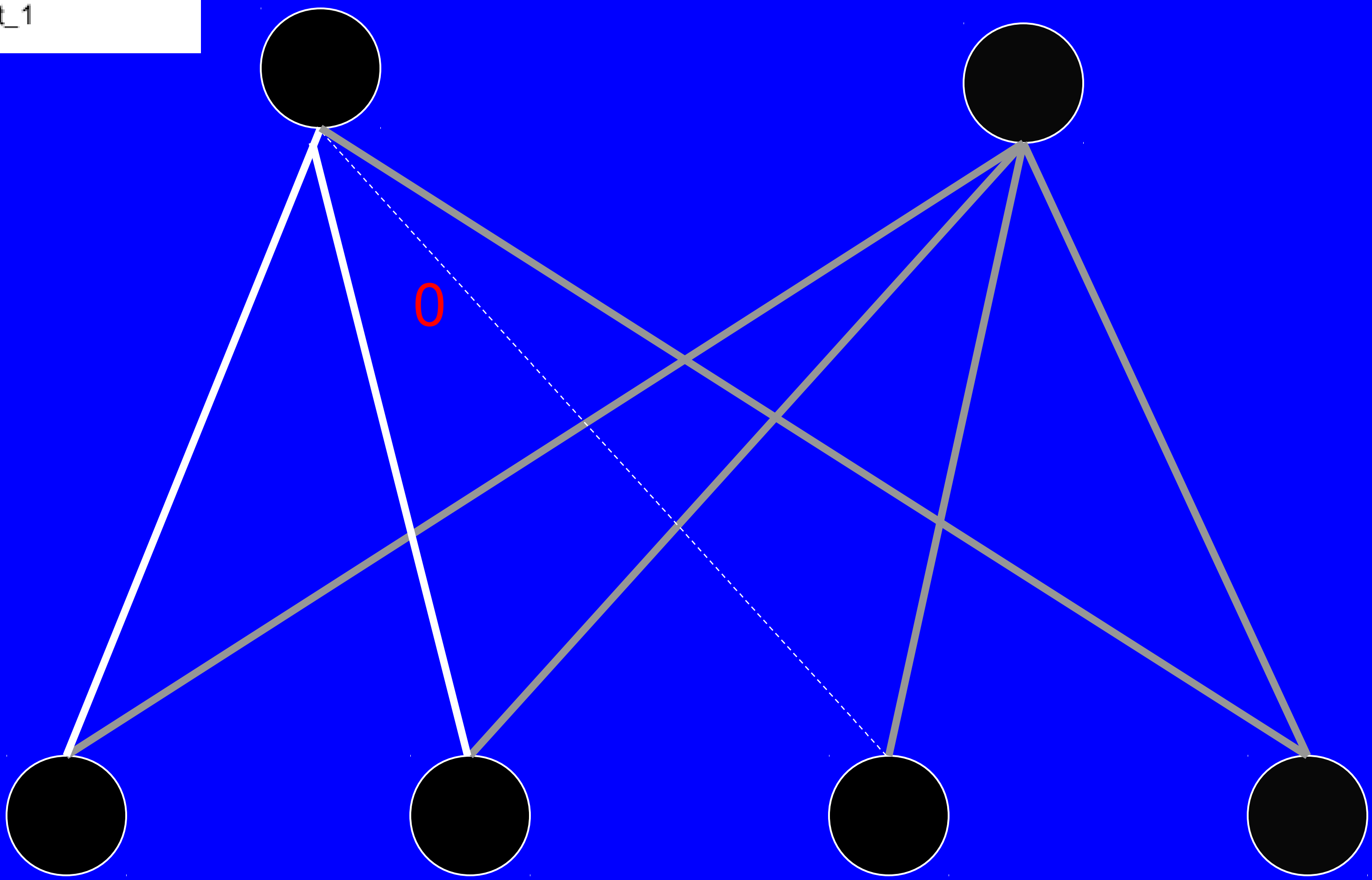

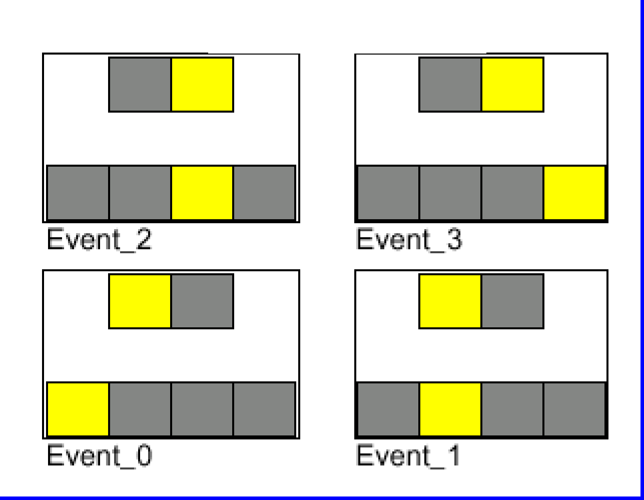

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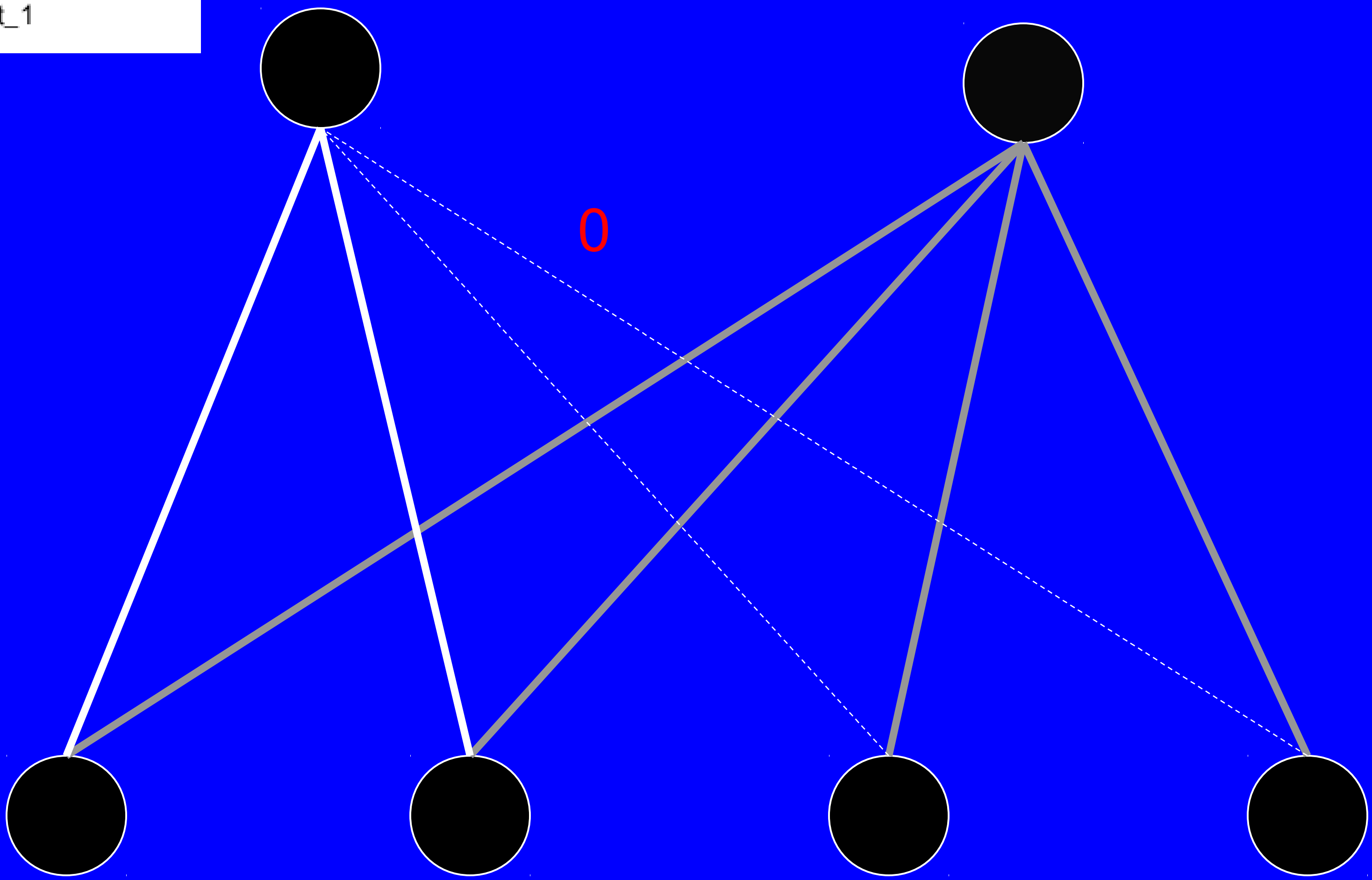

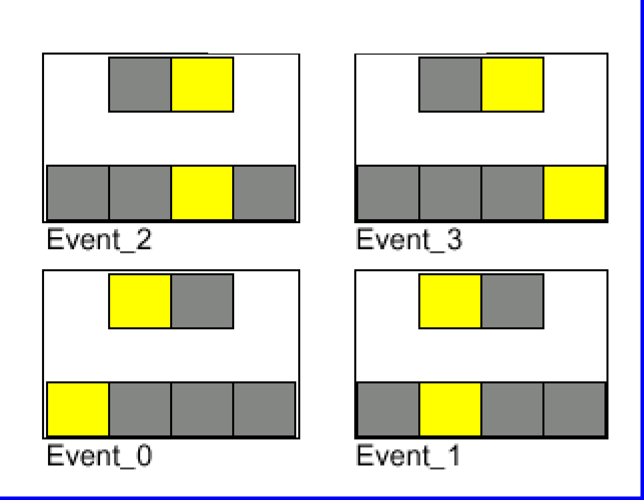

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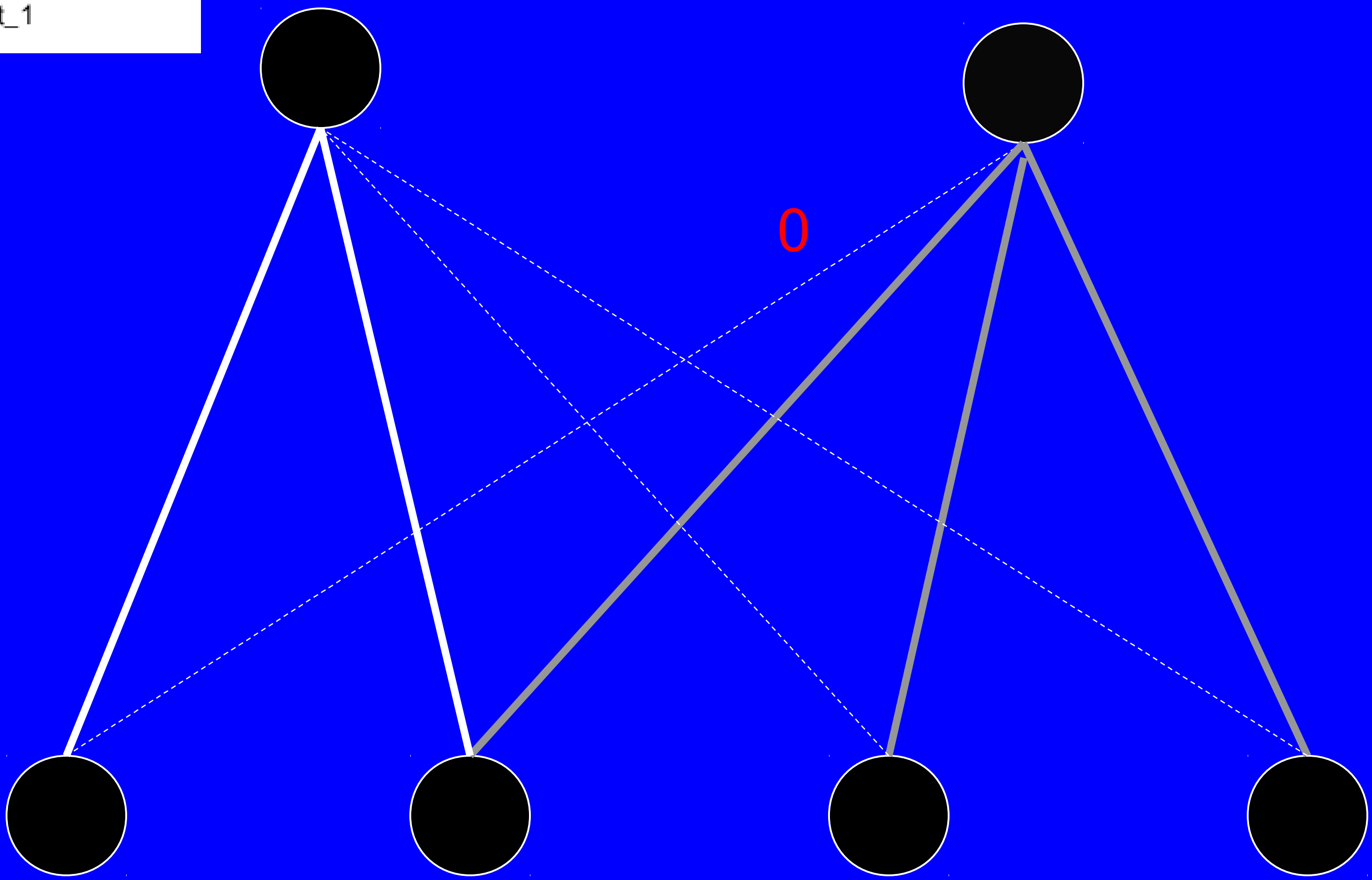

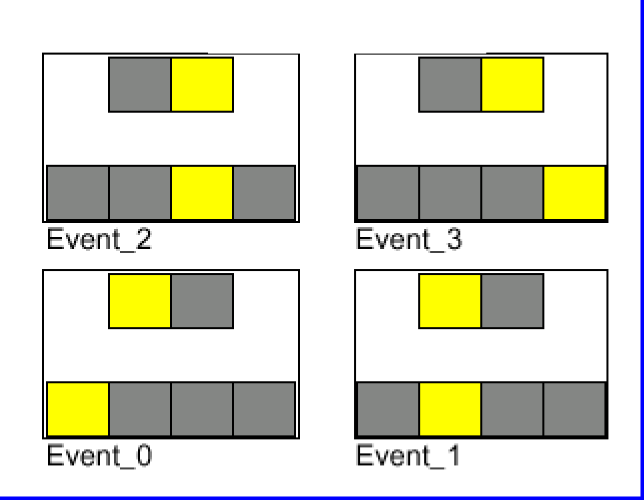

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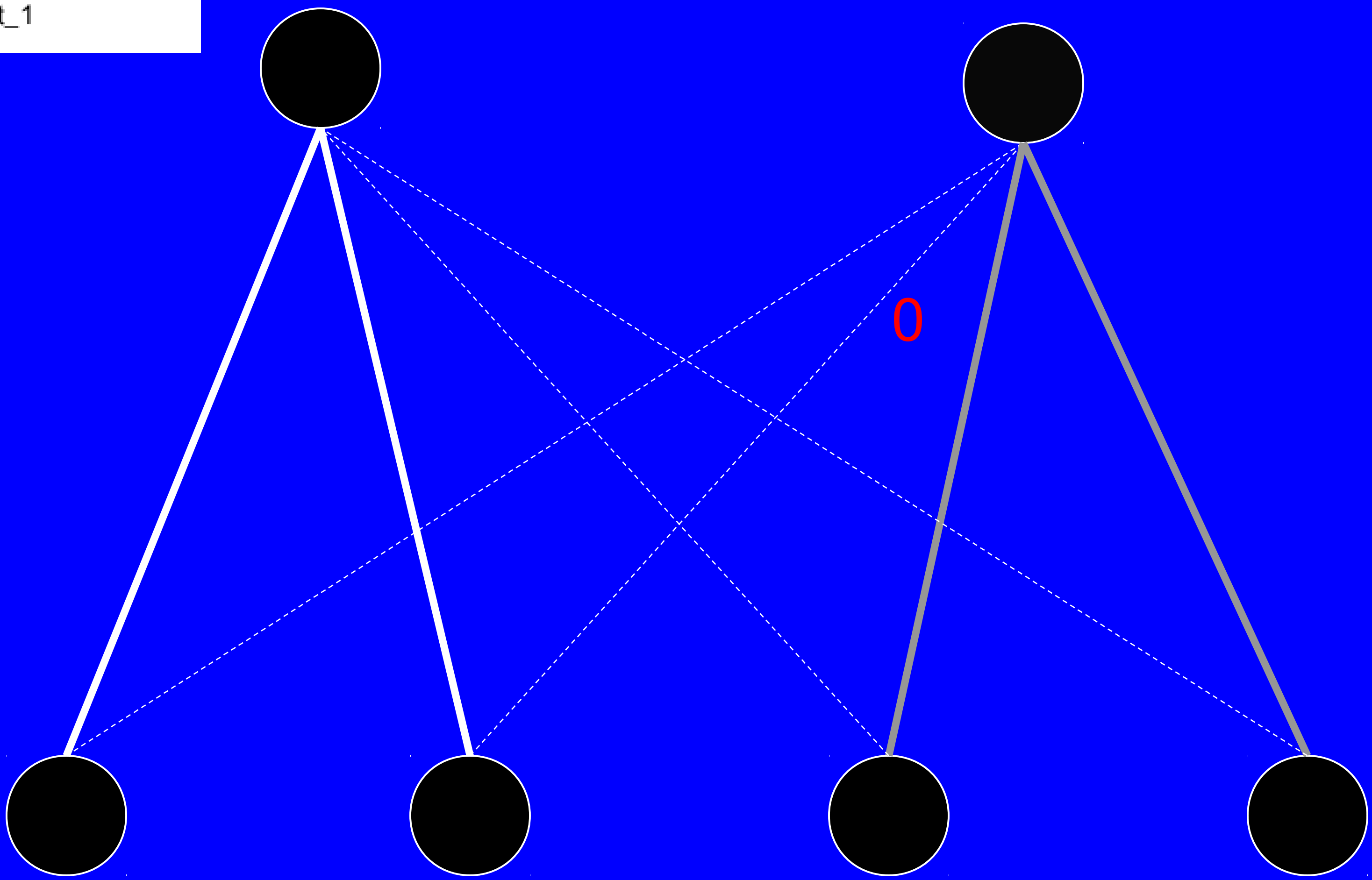

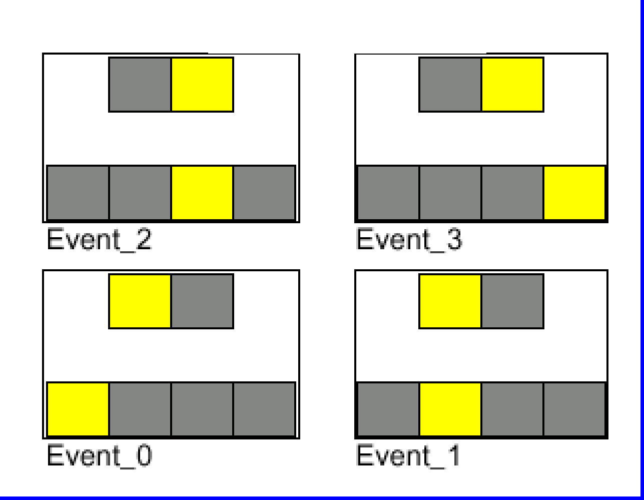

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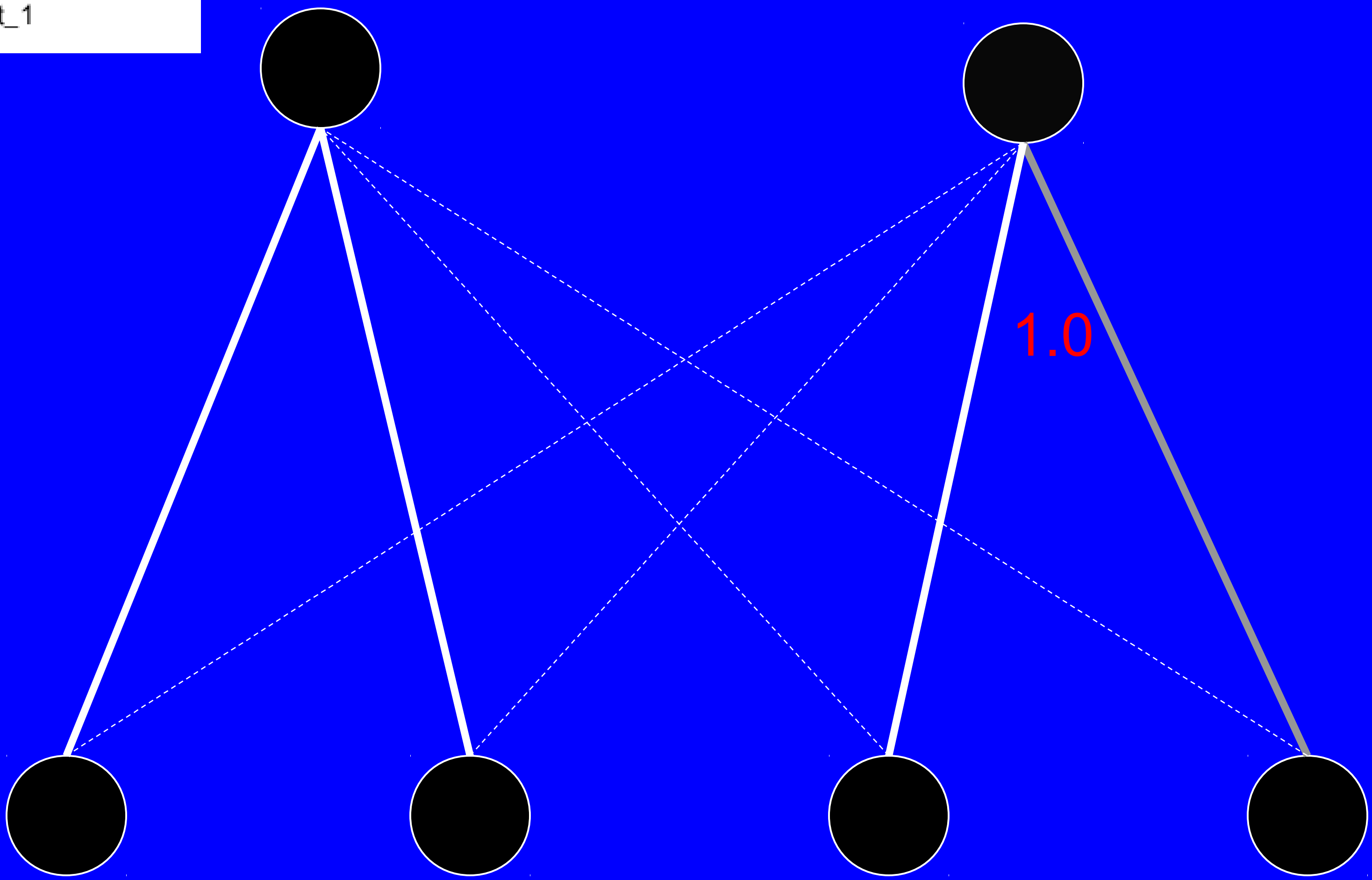

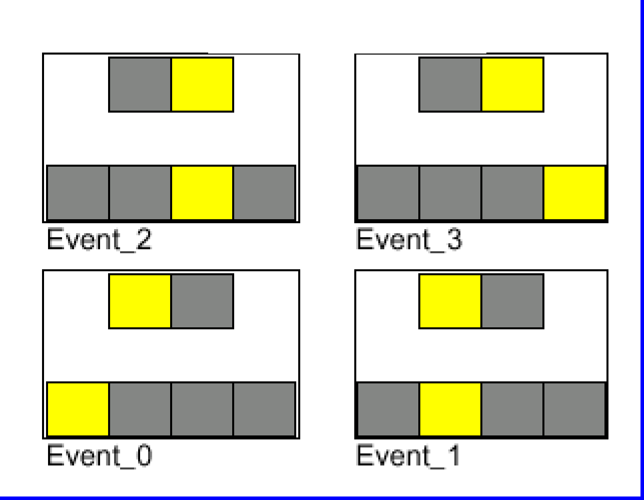

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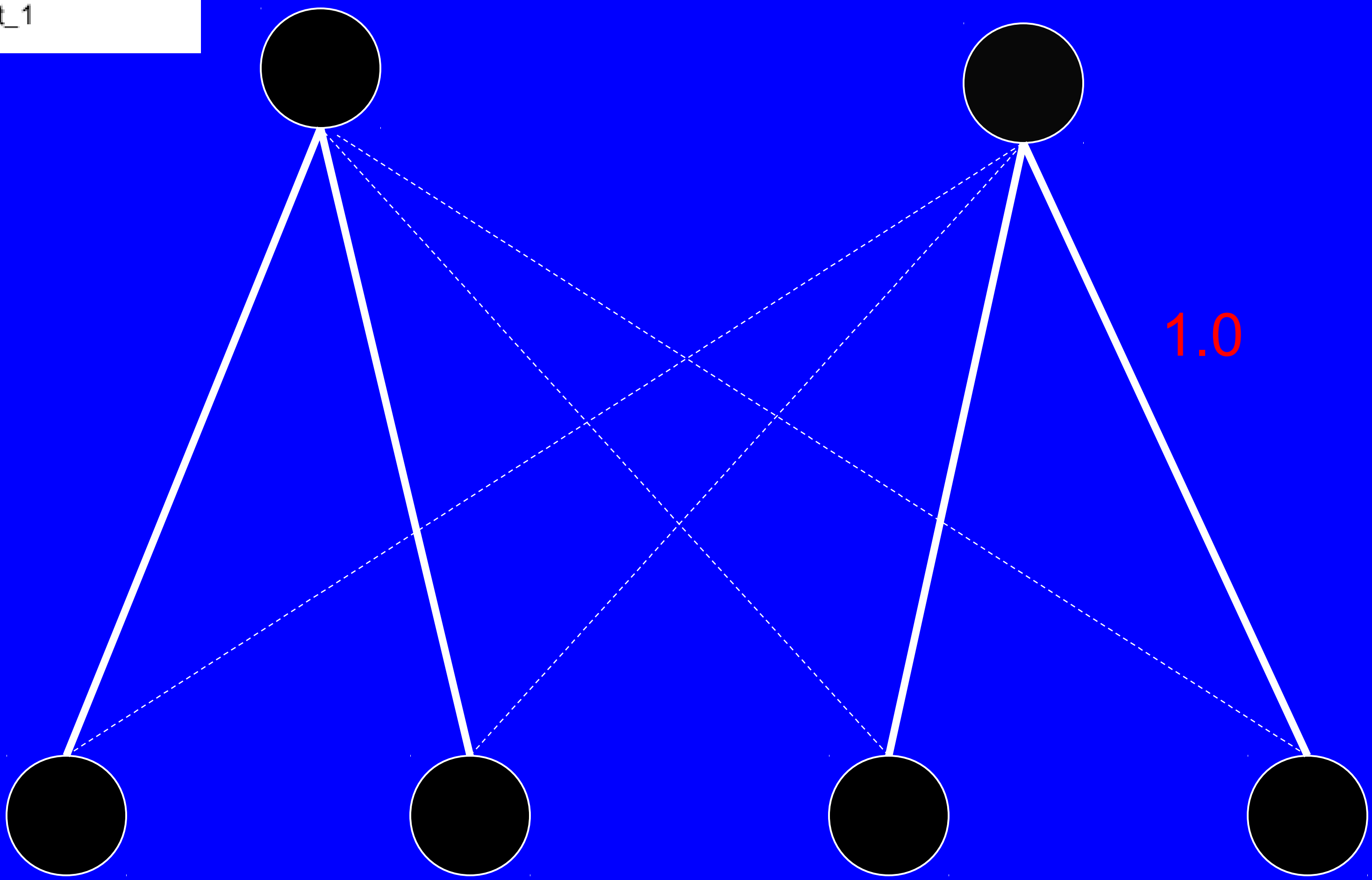
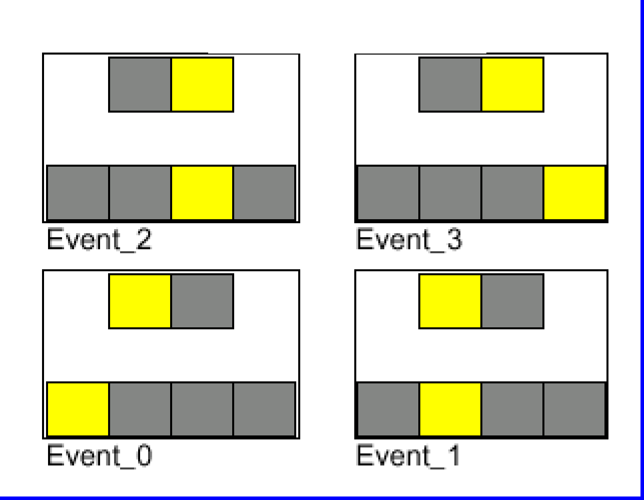
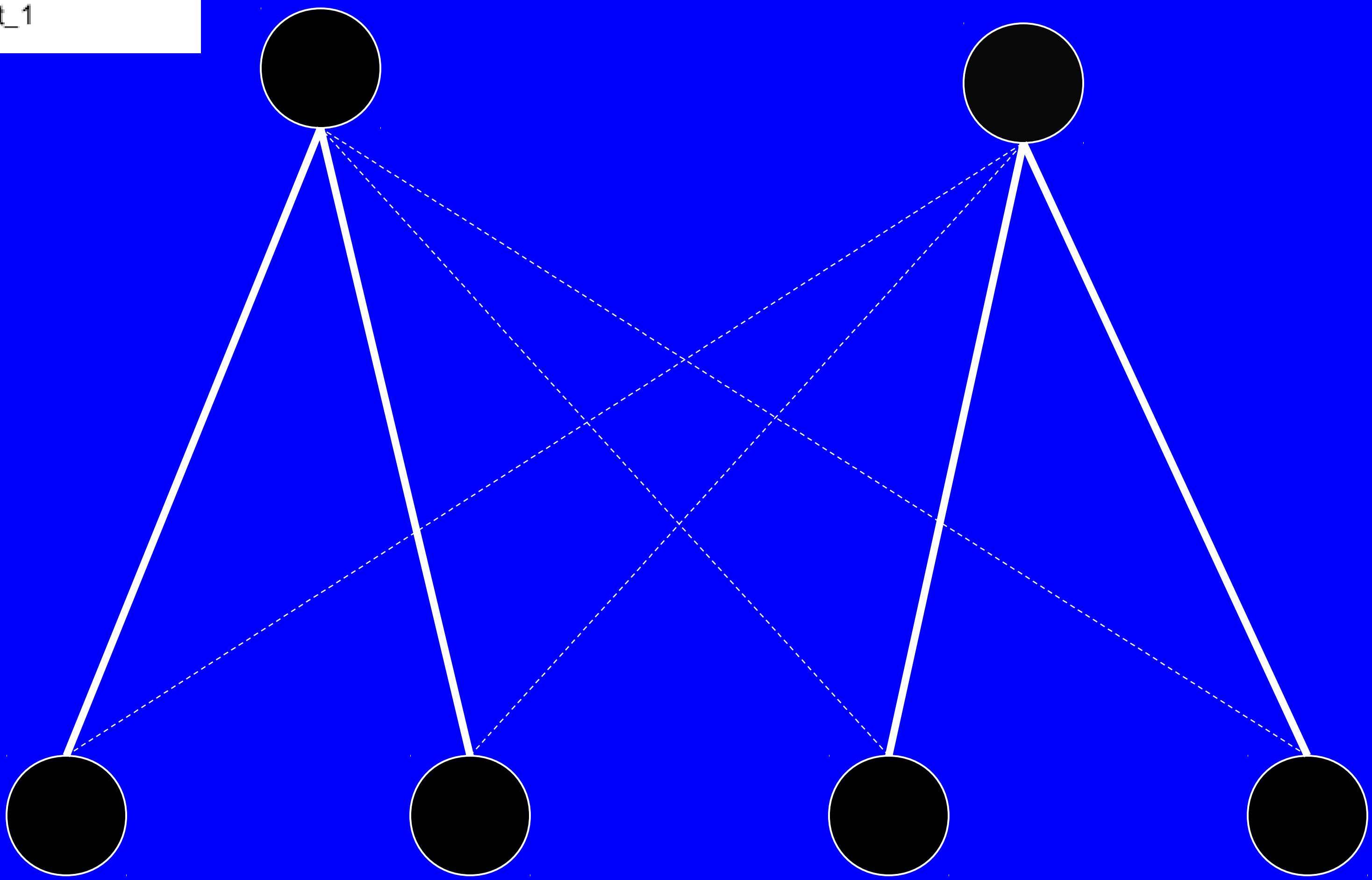


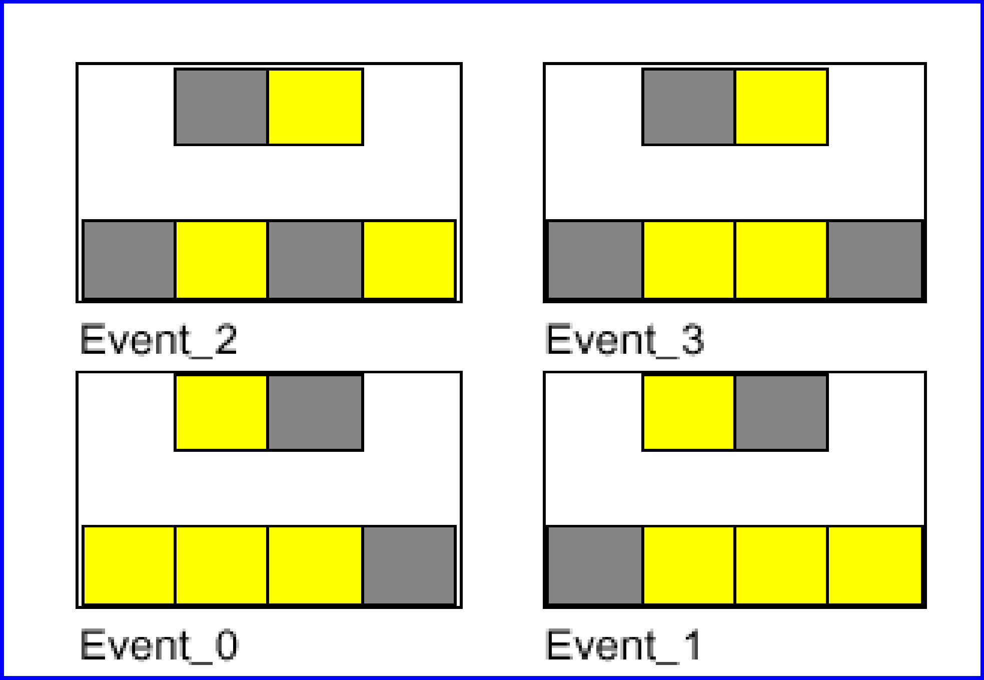


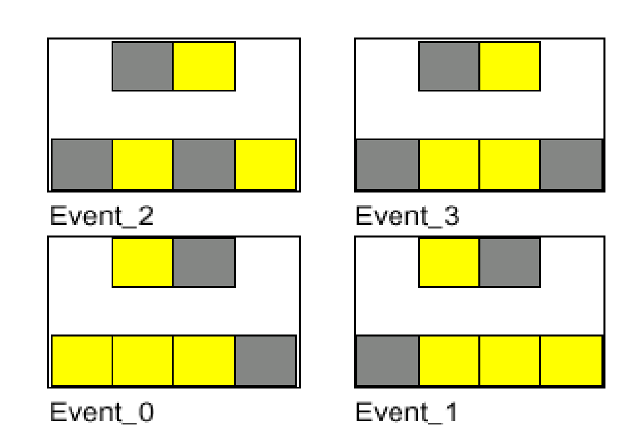
Hebb can solve the task!



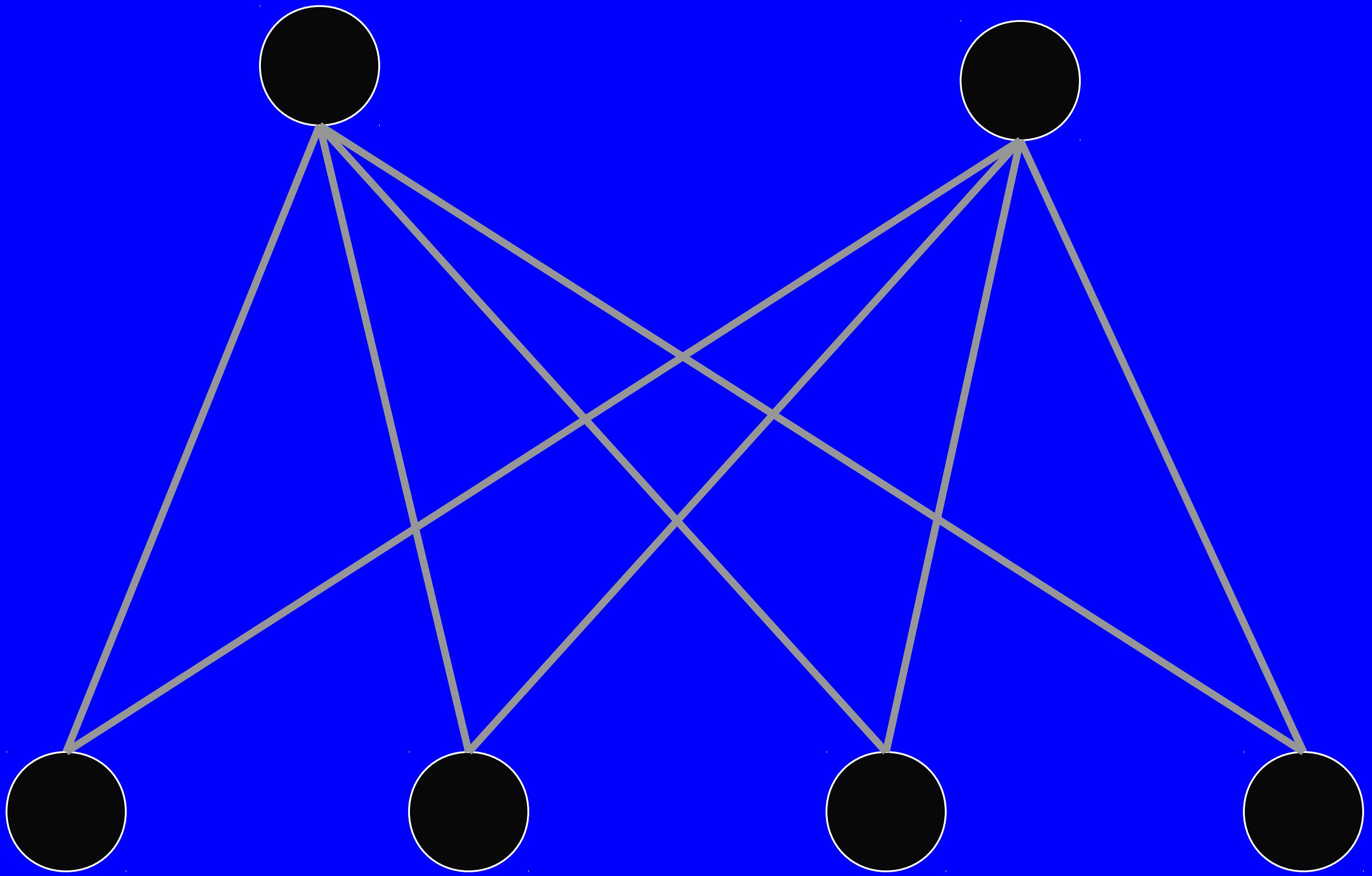
Another (Harder) Mapping

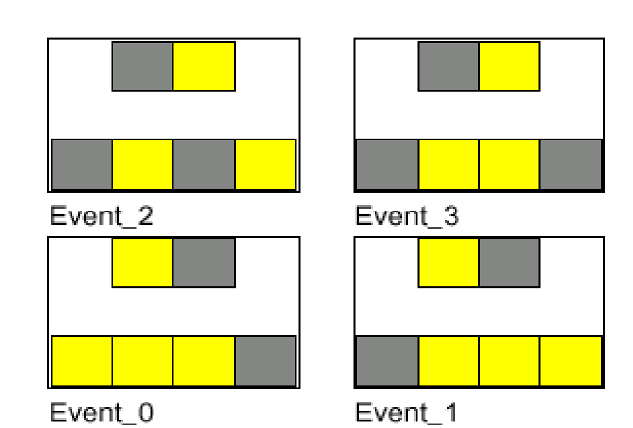
- overlap between inputs
- input units associated with multiple outputs





The mapping is solvable!





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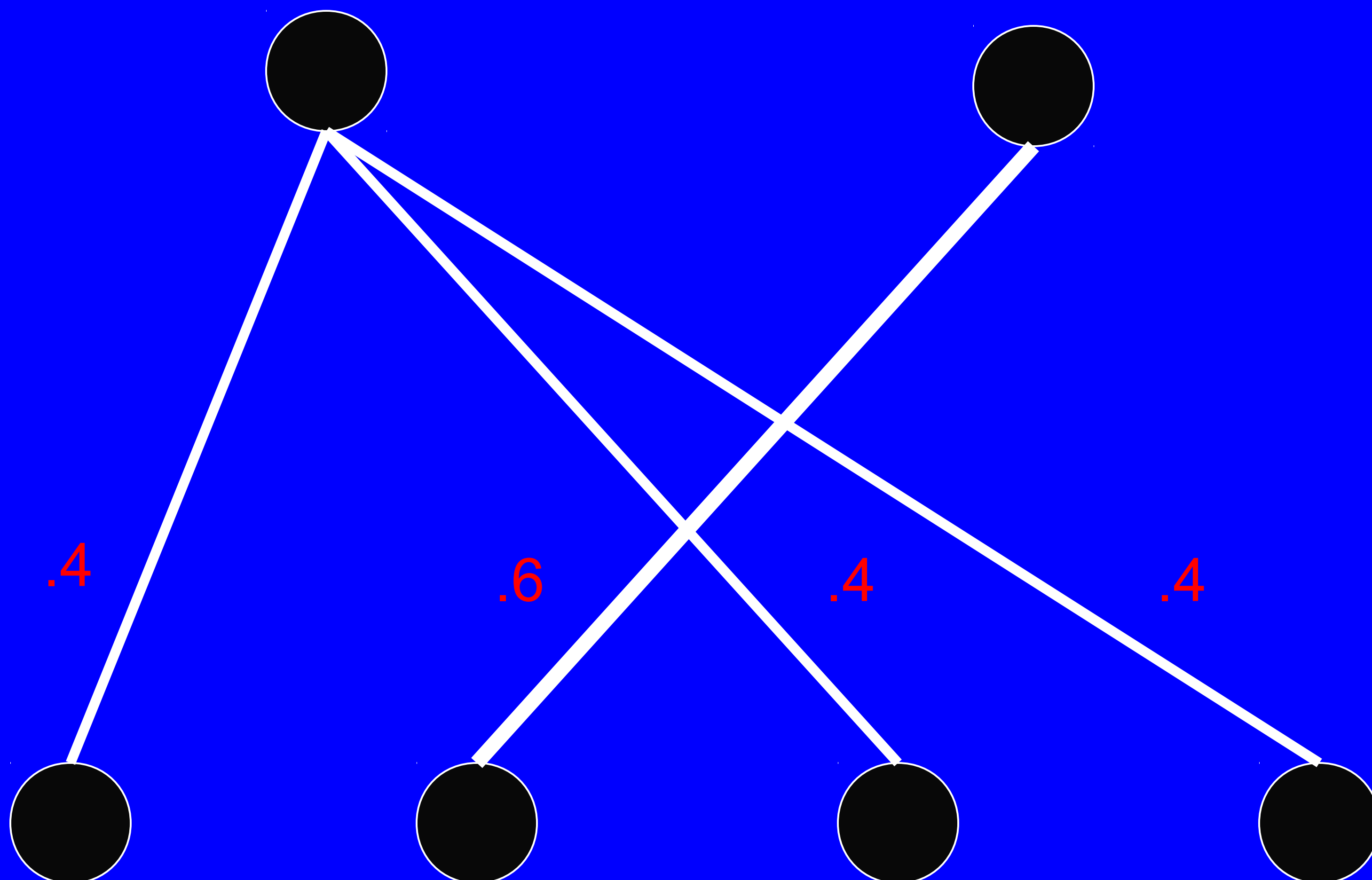

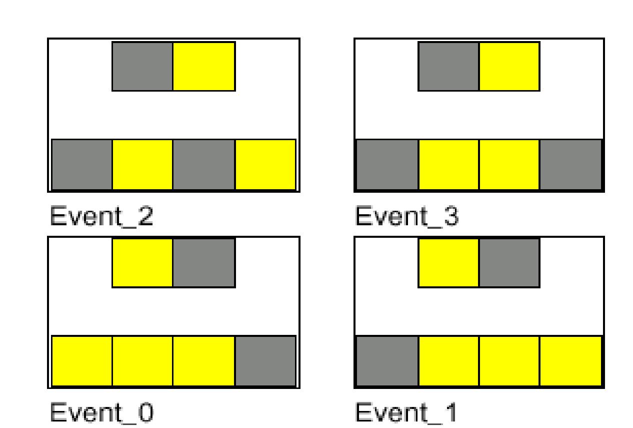

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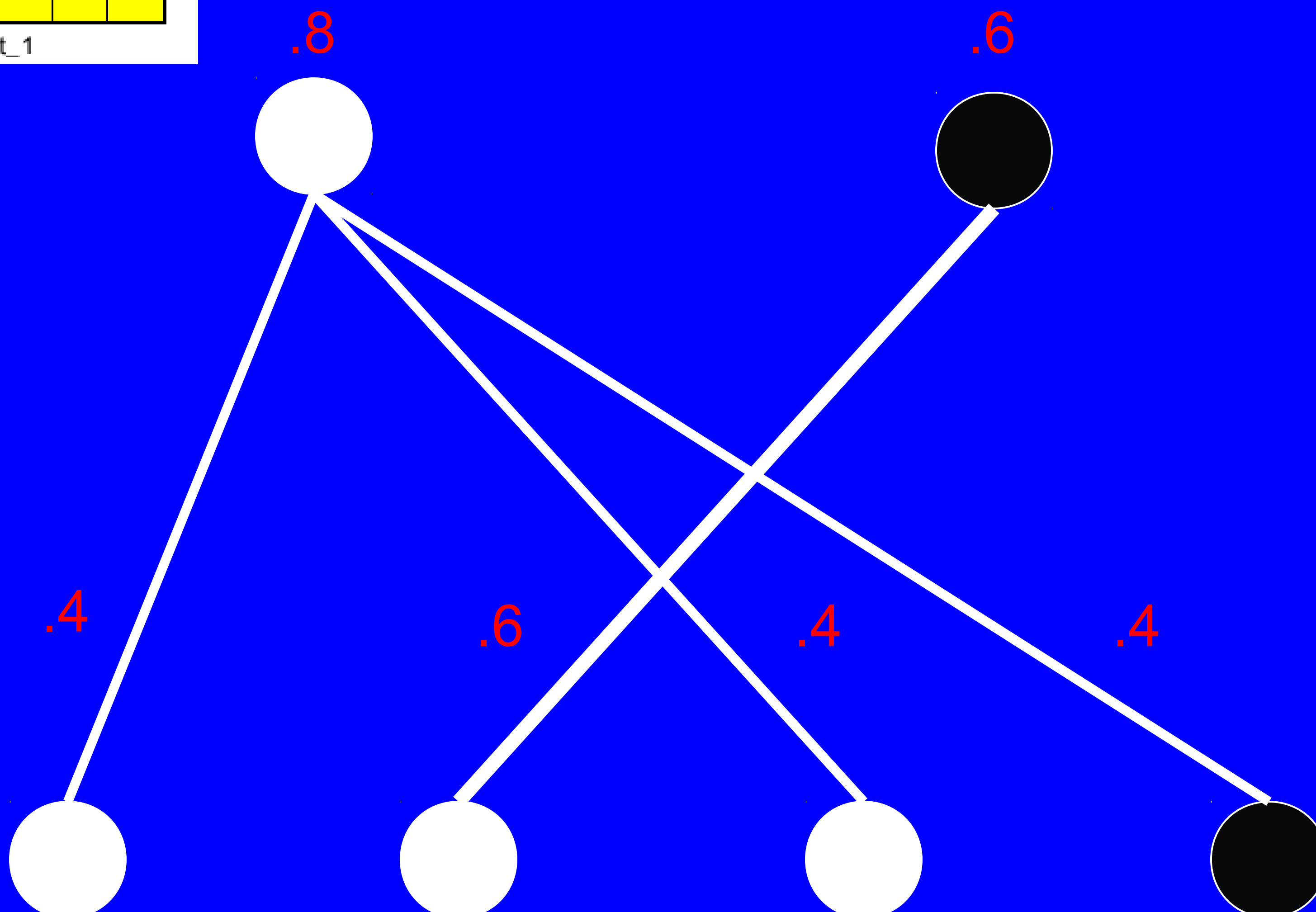

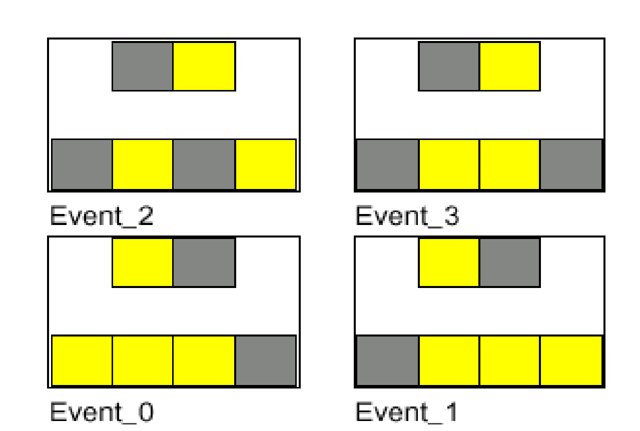

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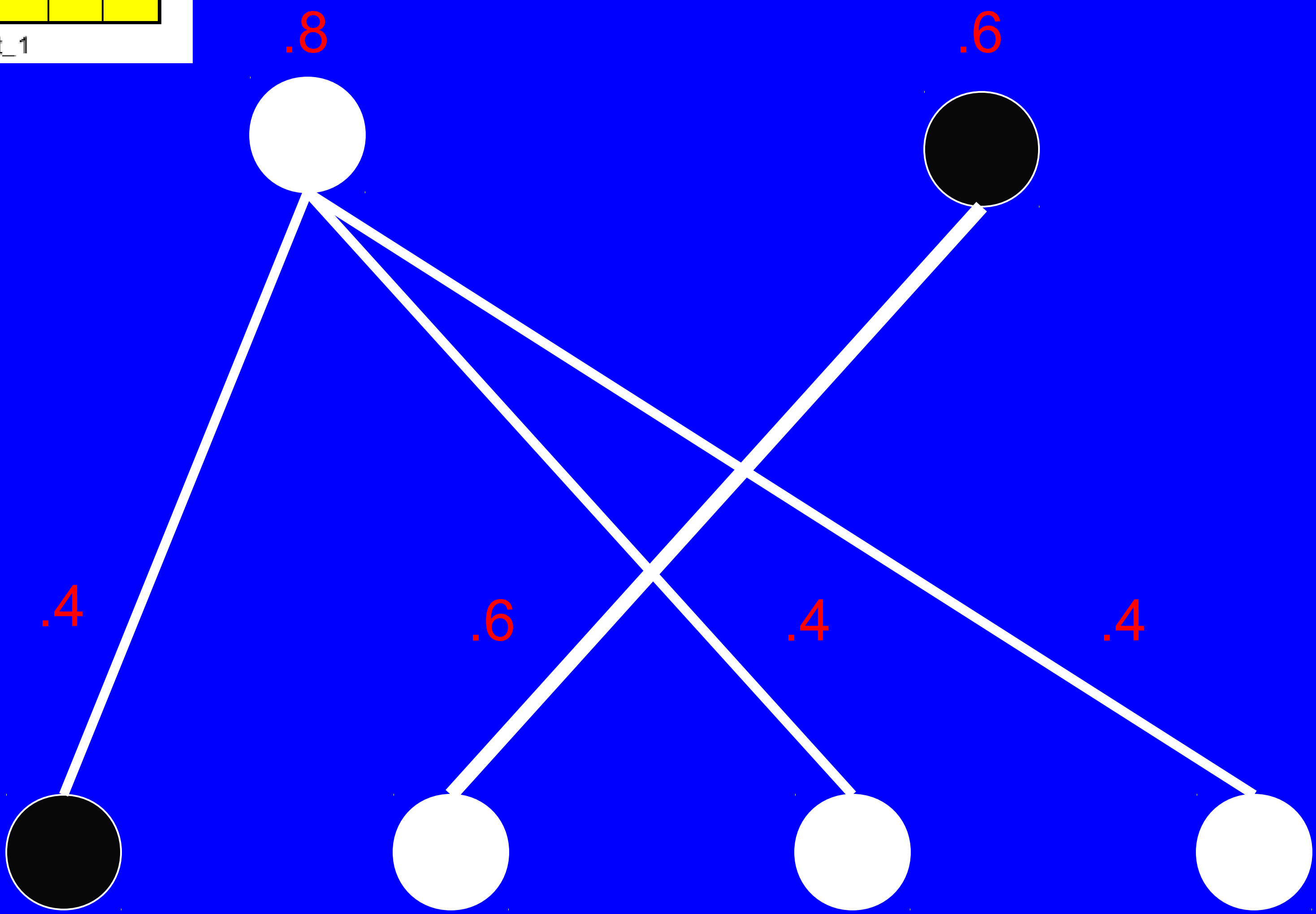

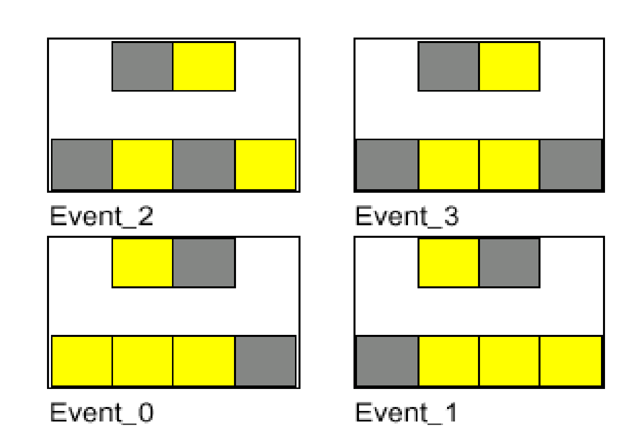

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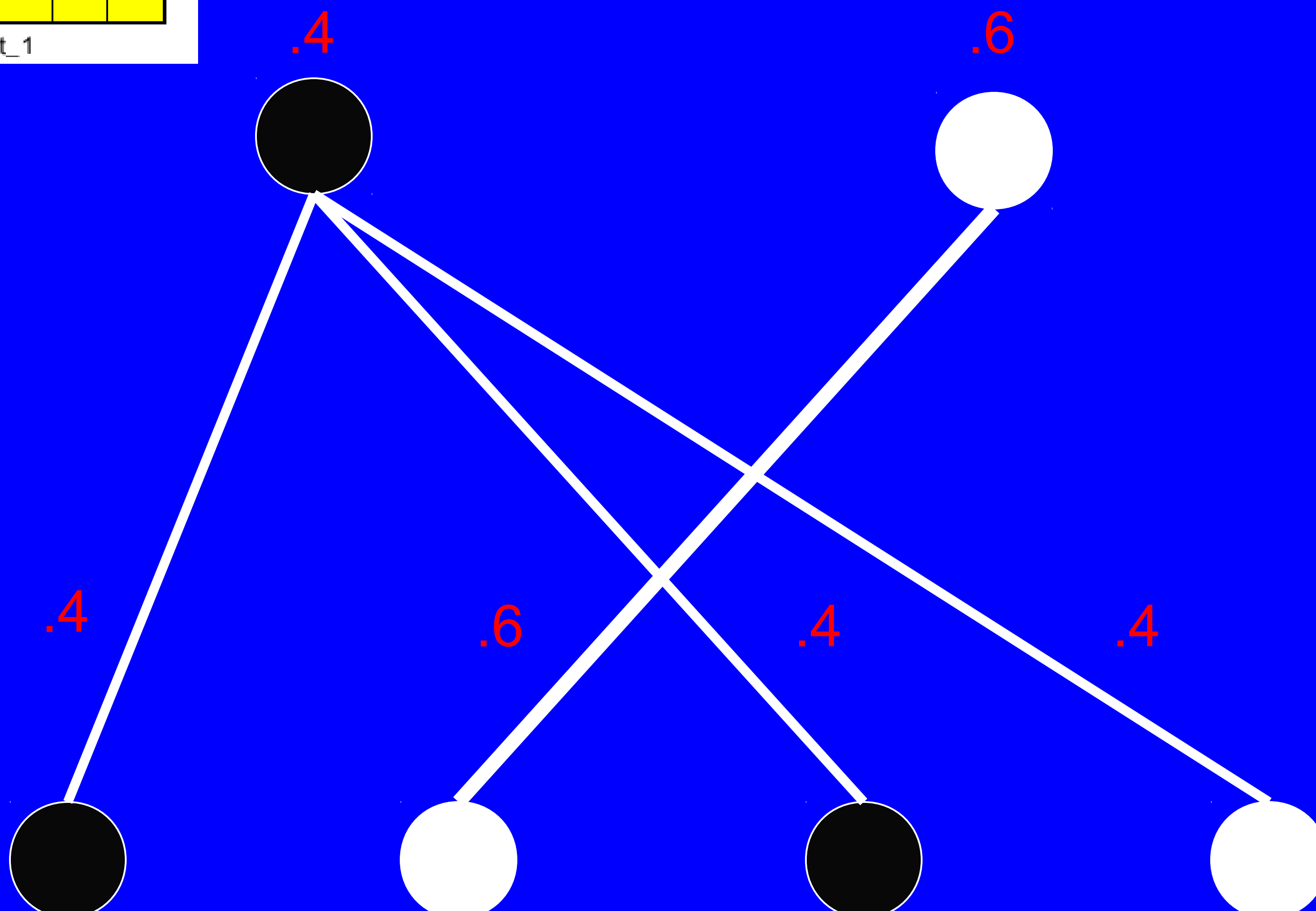

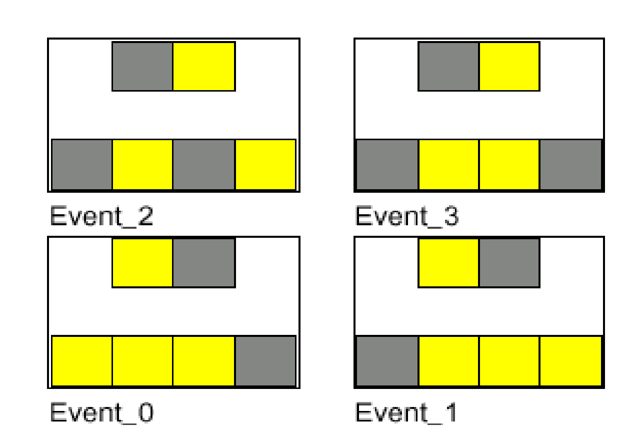

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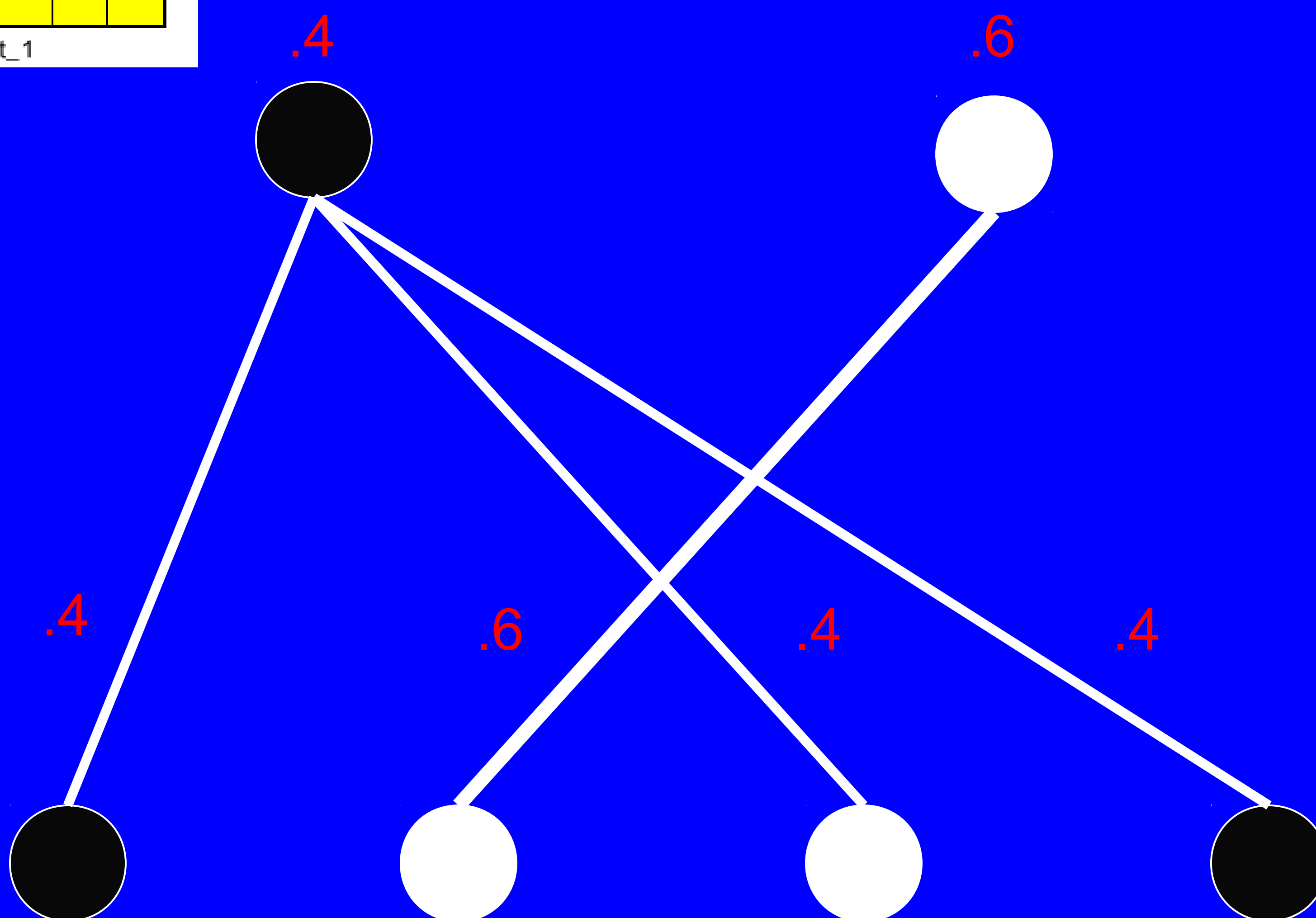

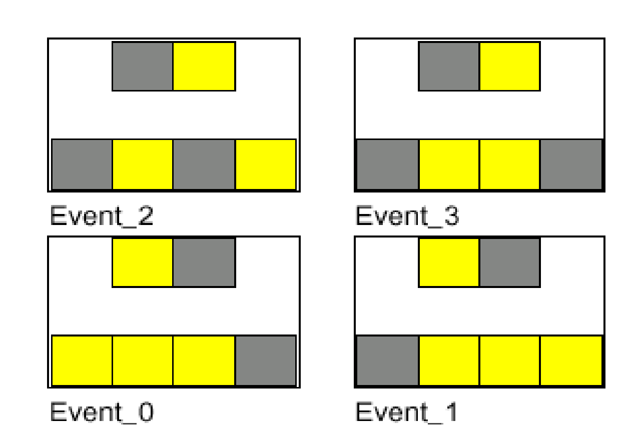

Hebbian learning:
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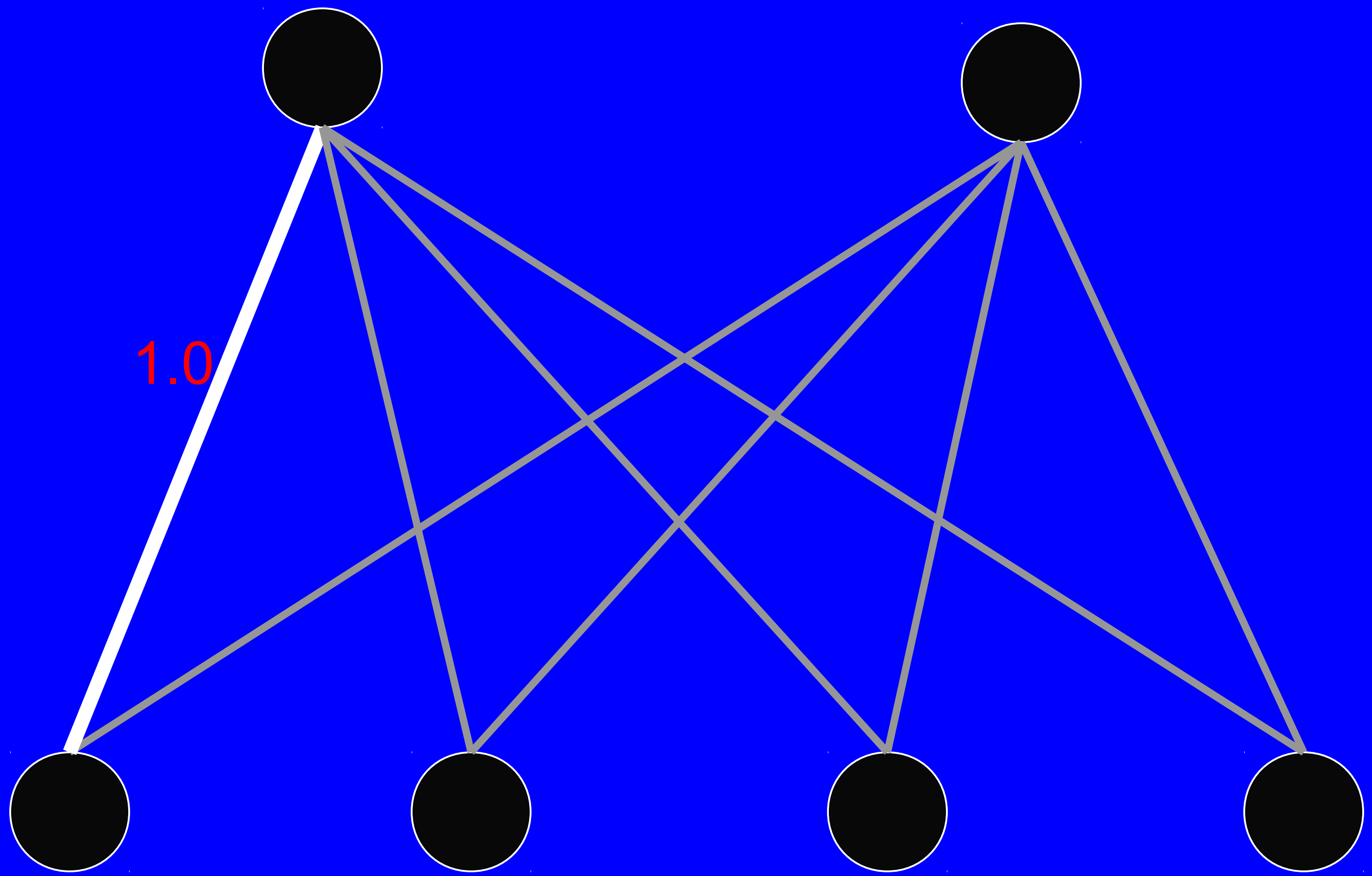

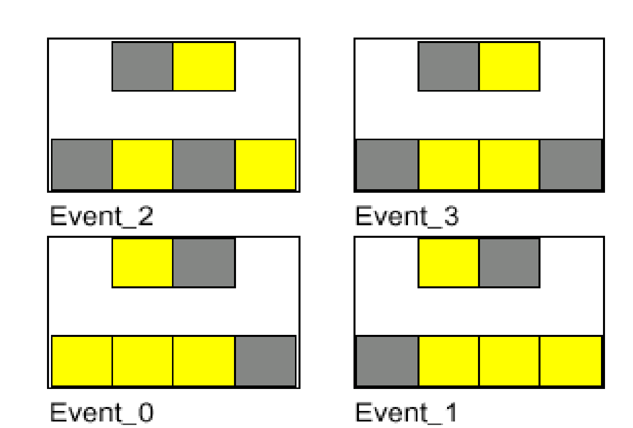

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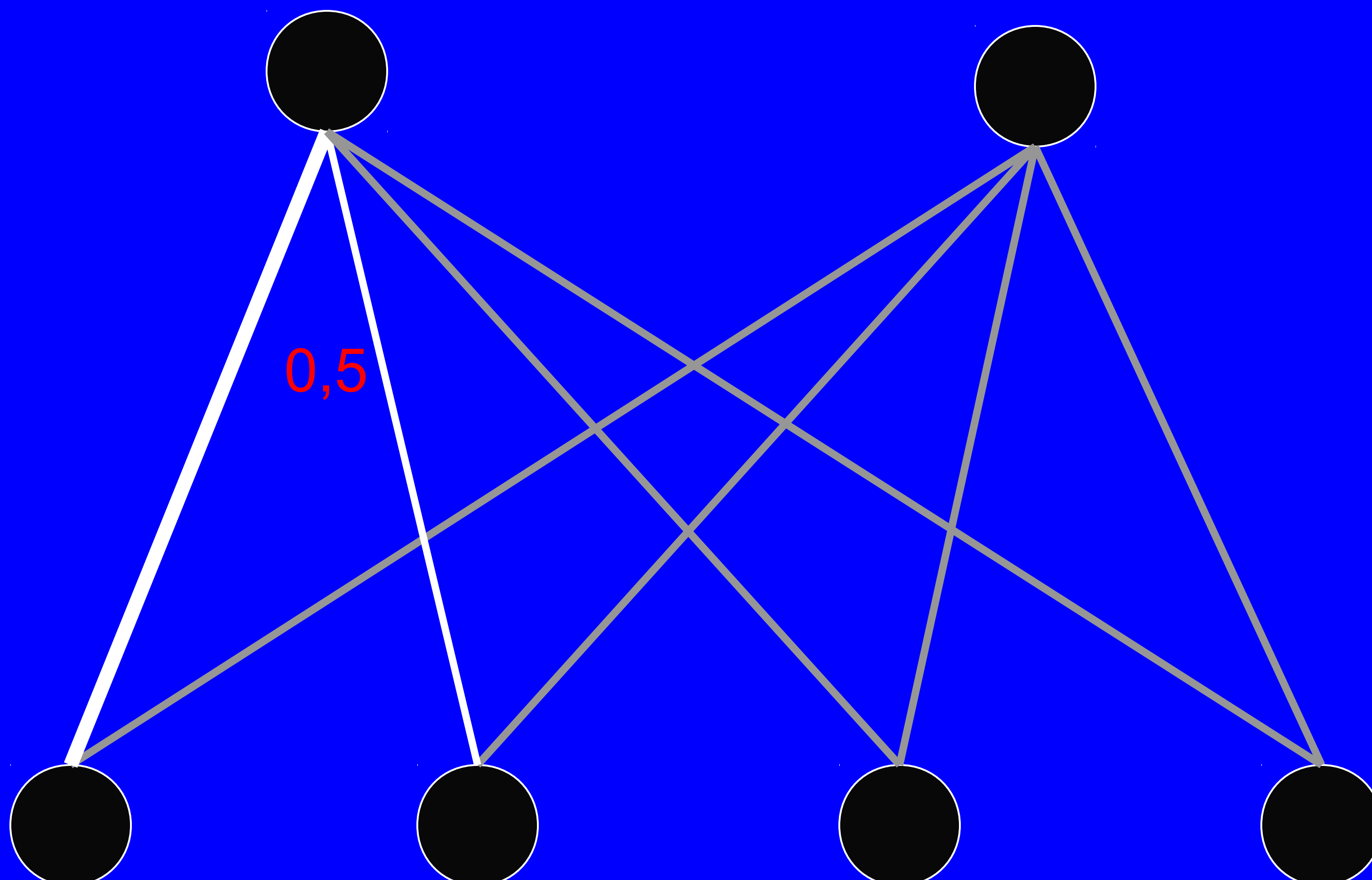

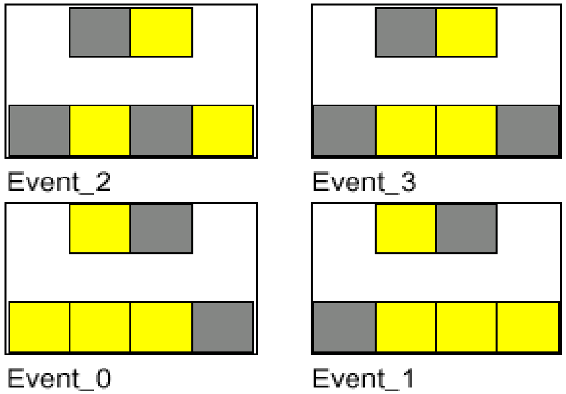

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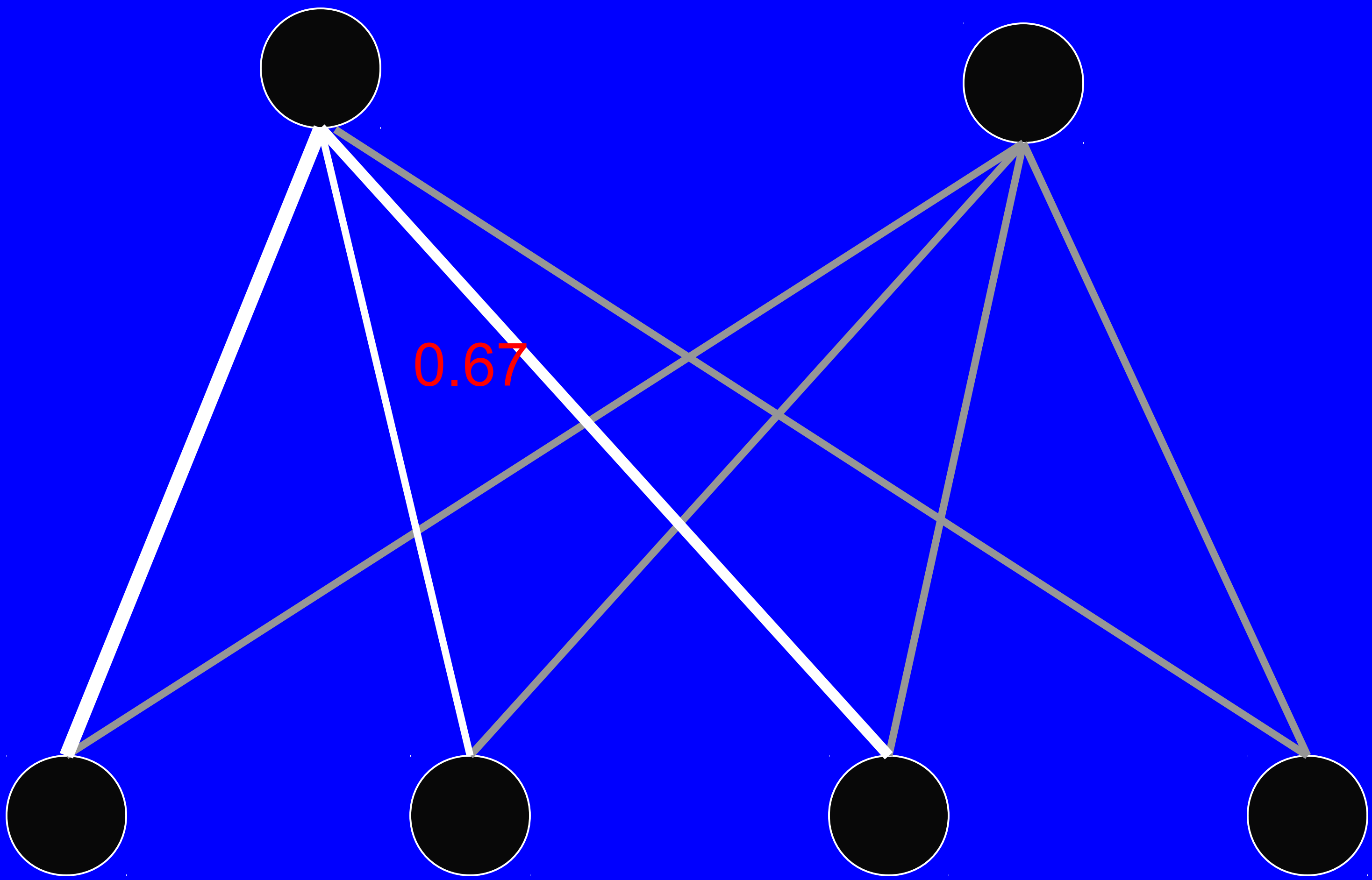

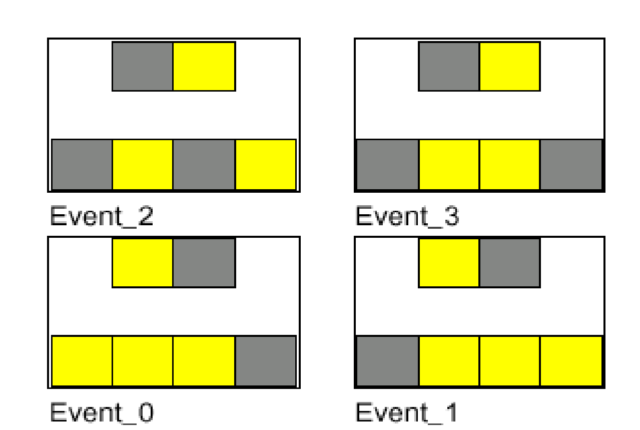

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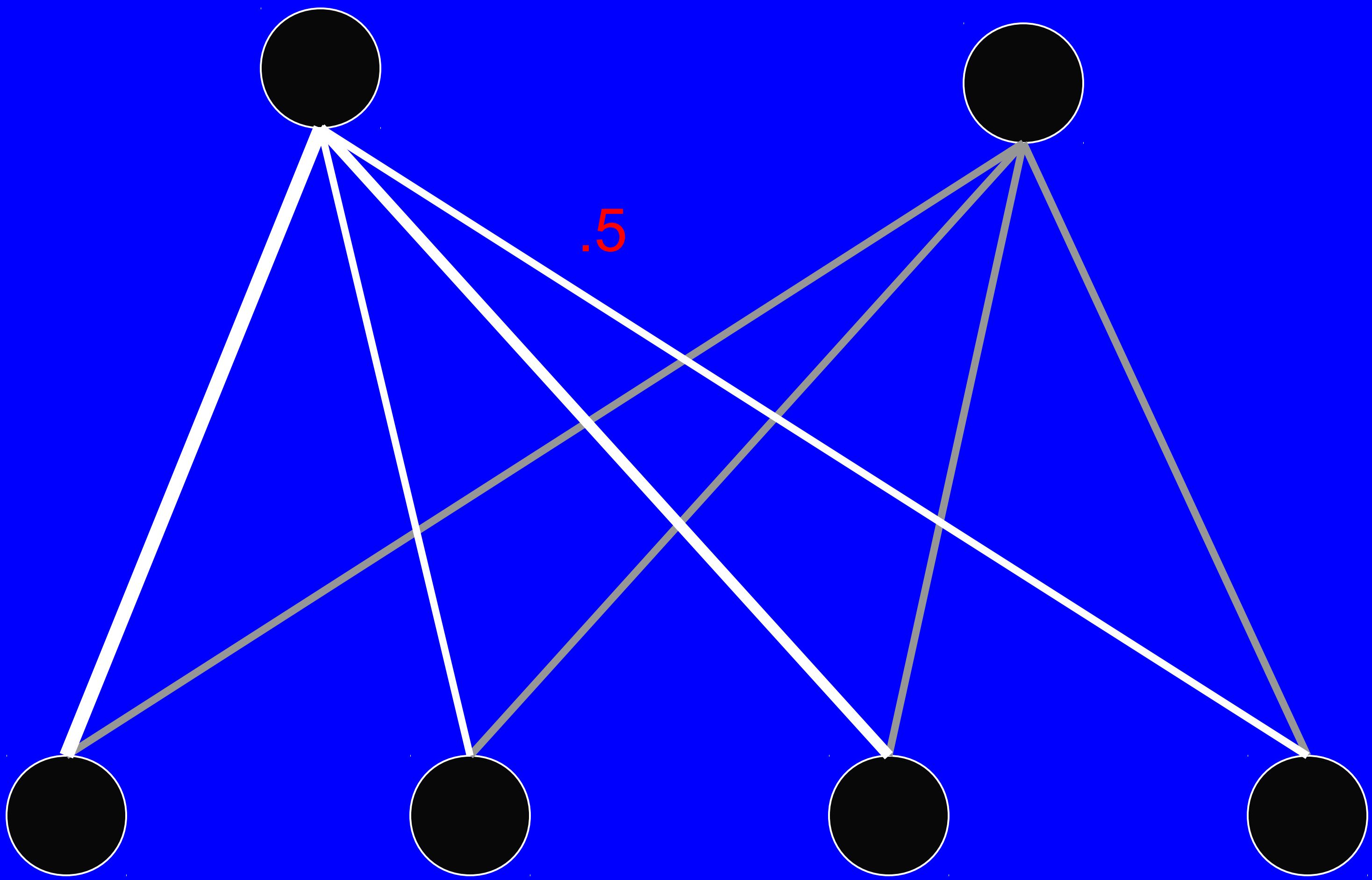

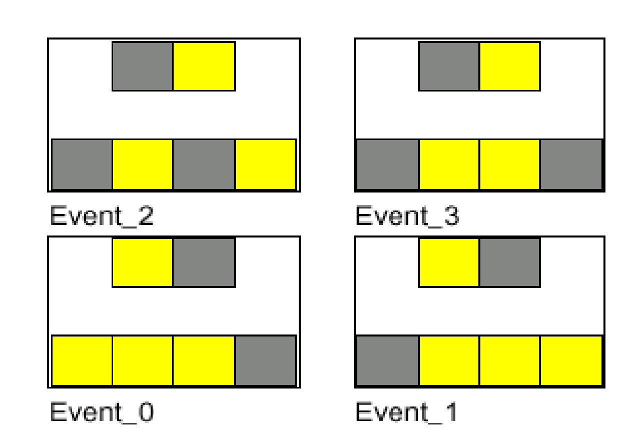

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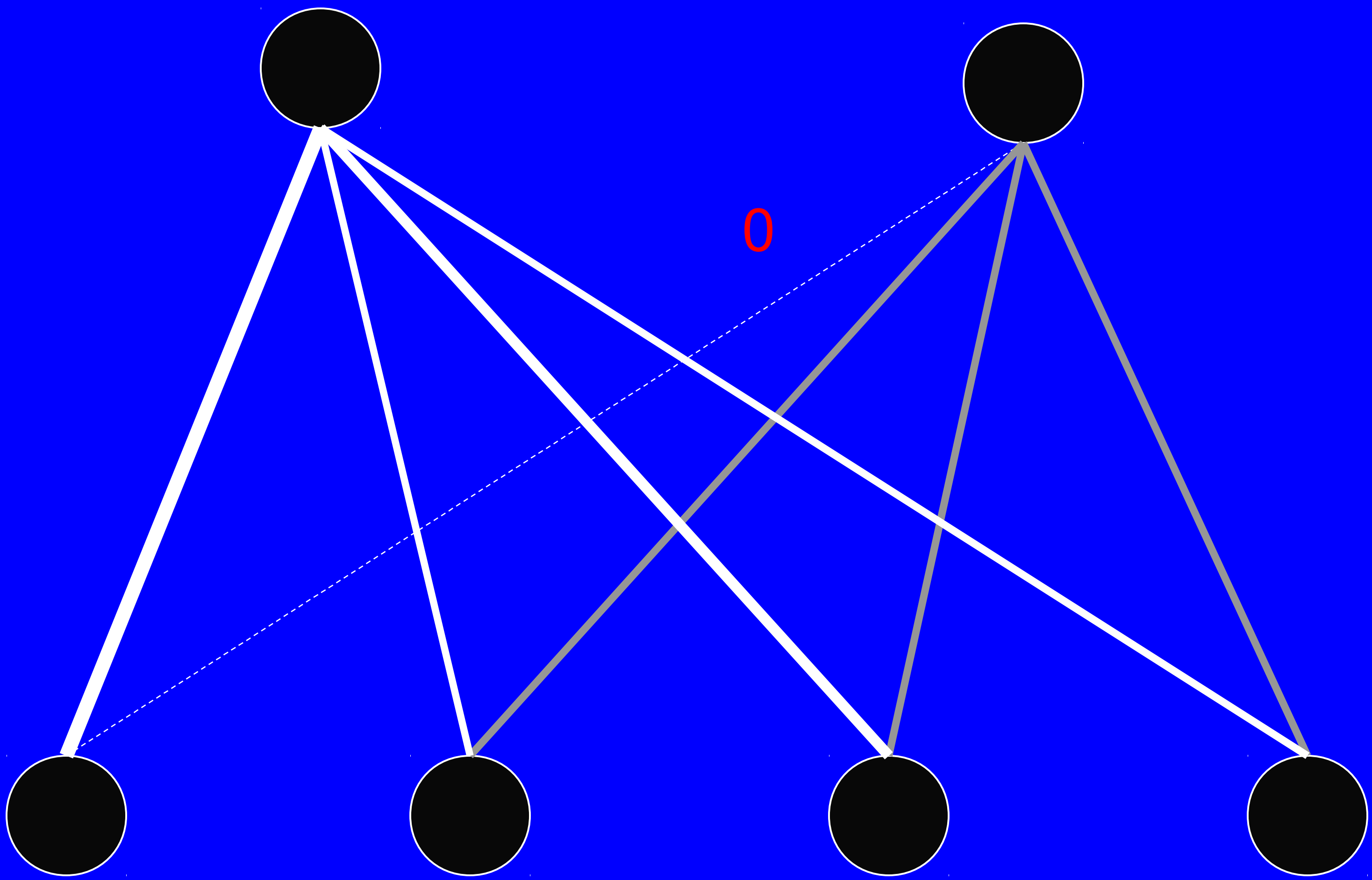

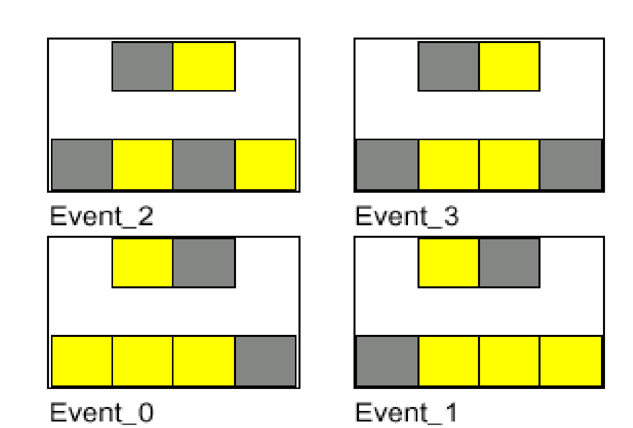

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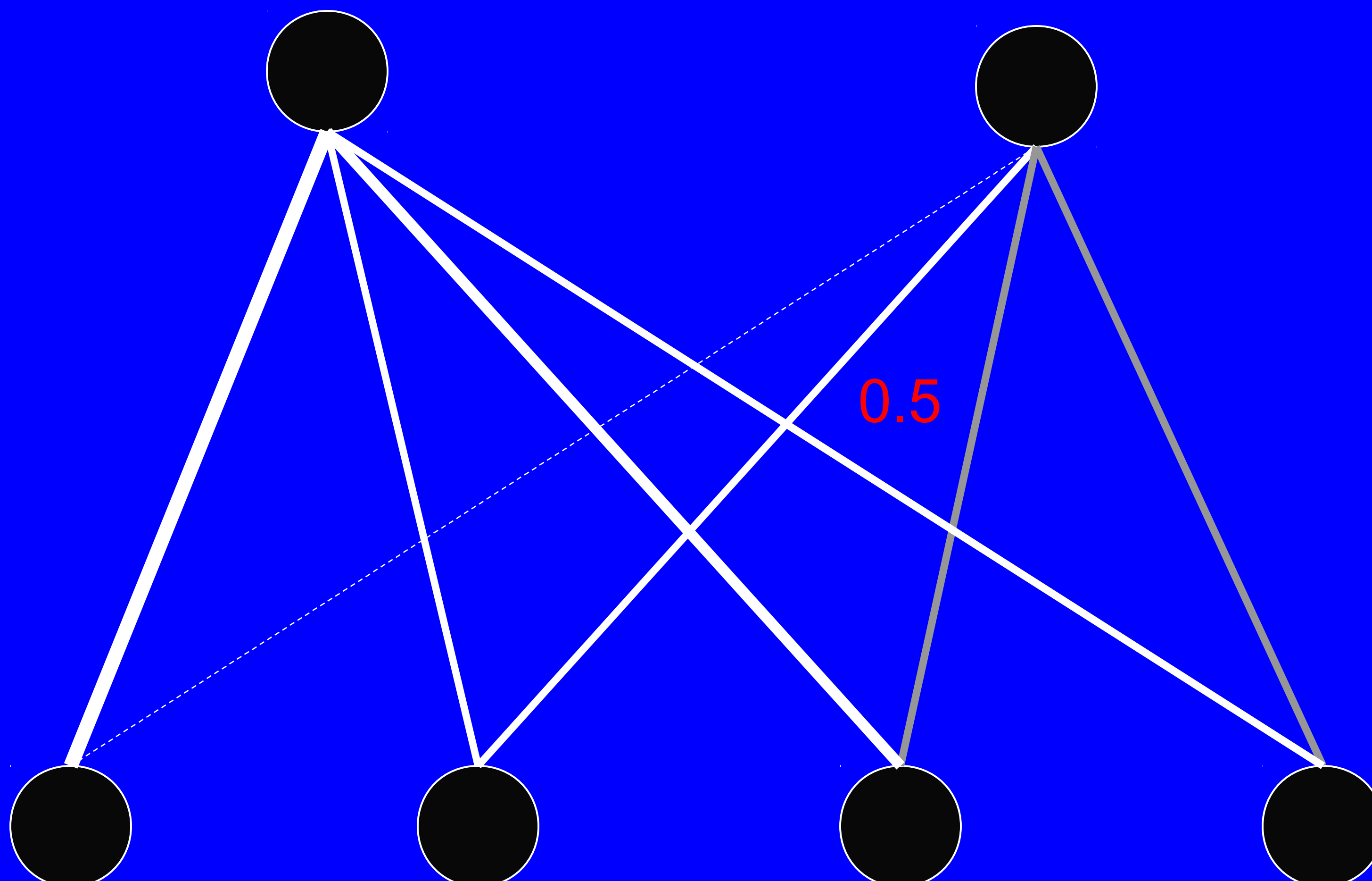

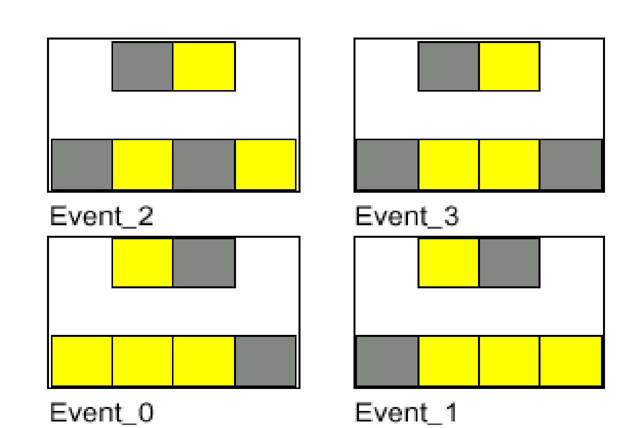

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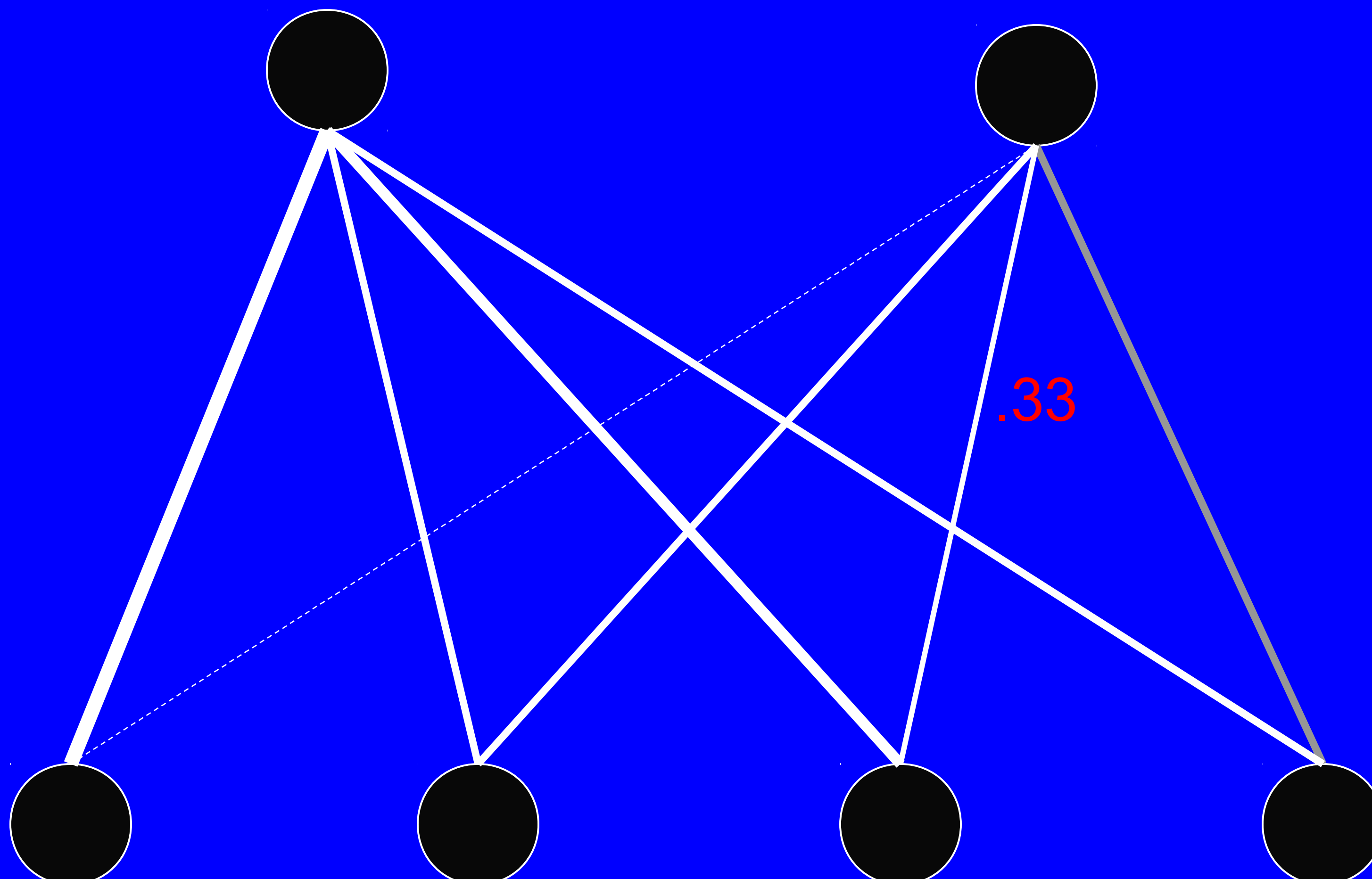

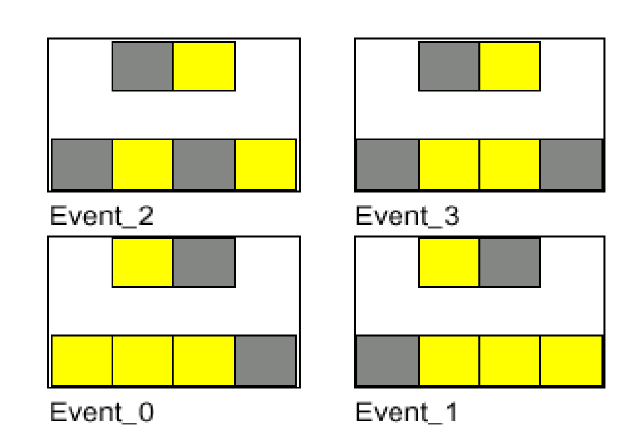

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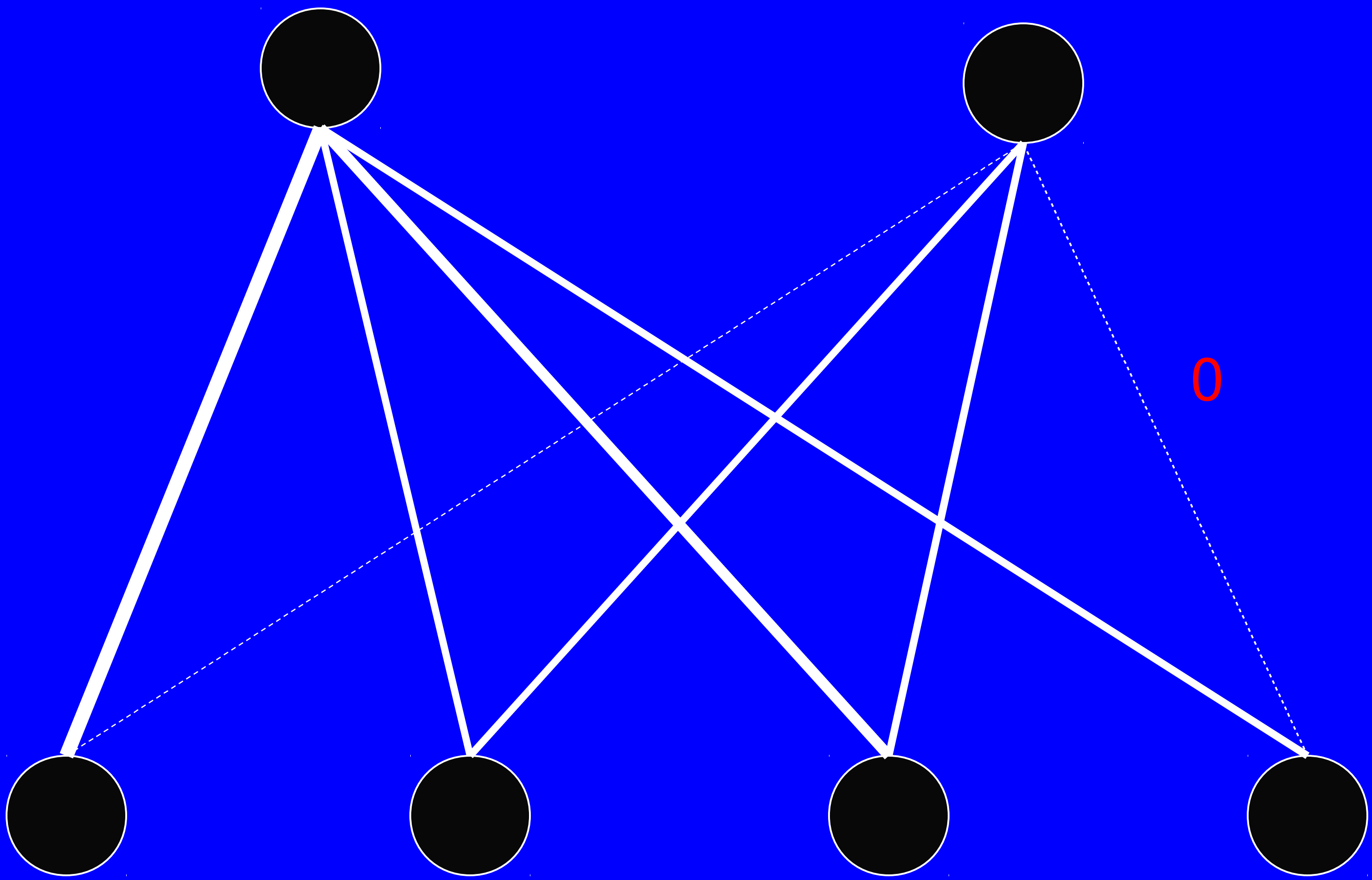
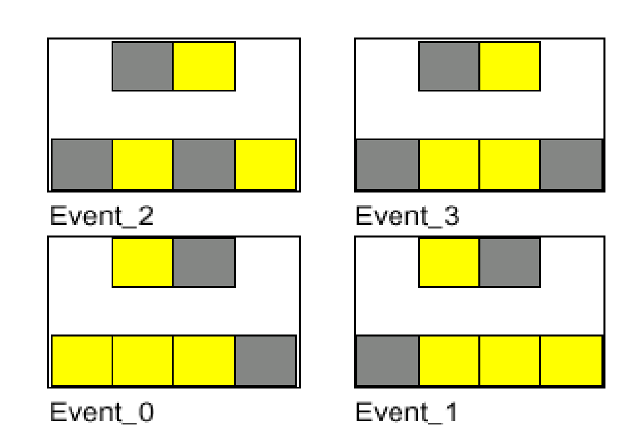
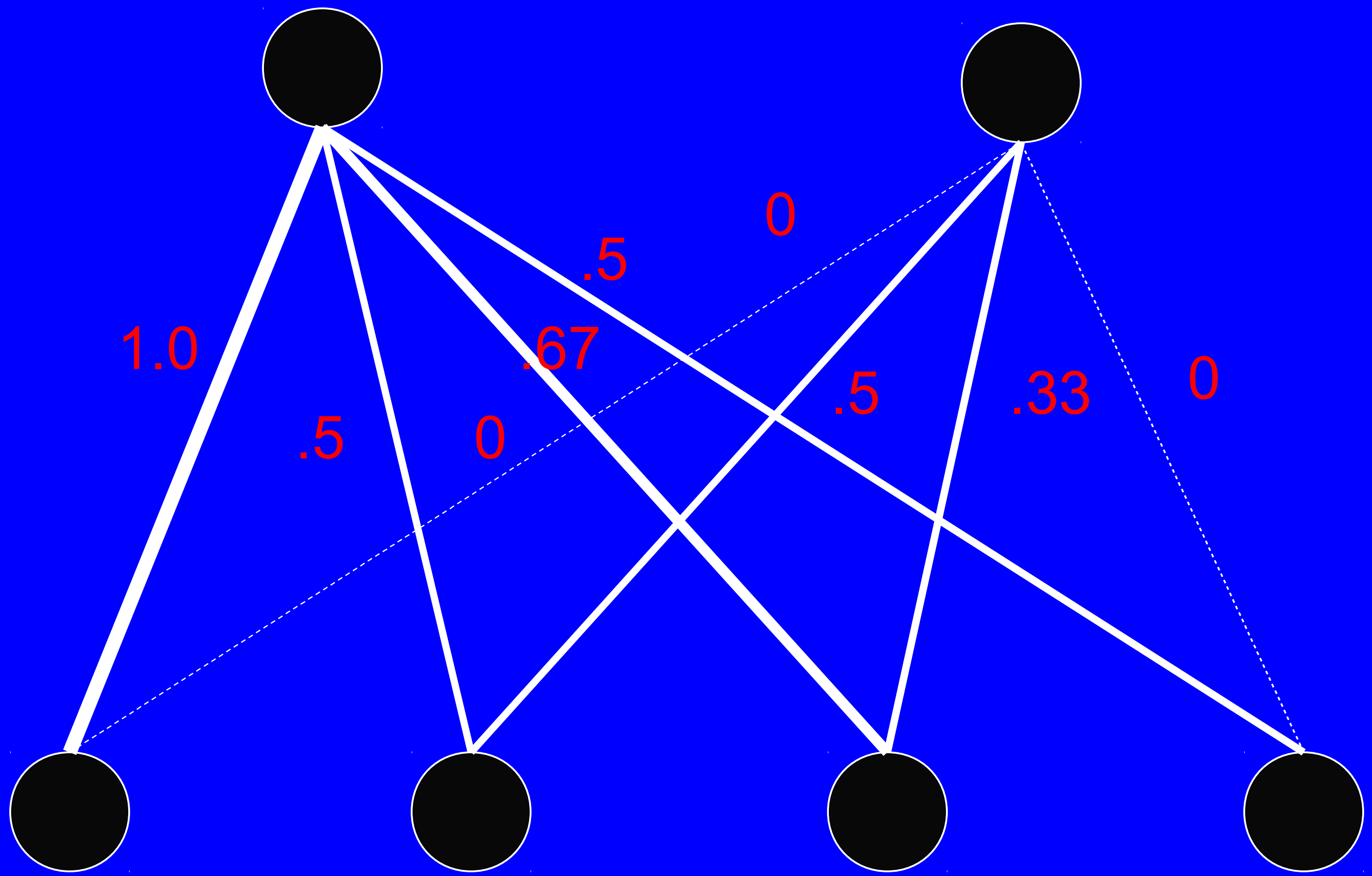
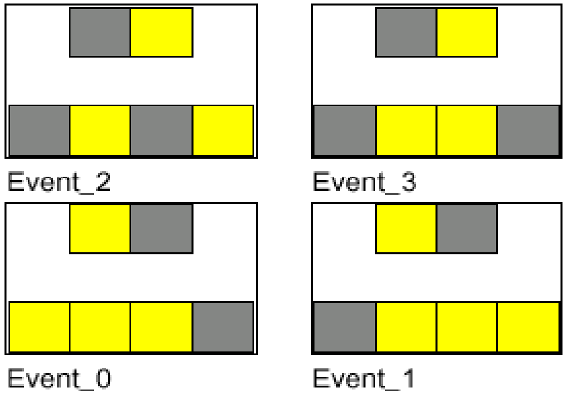


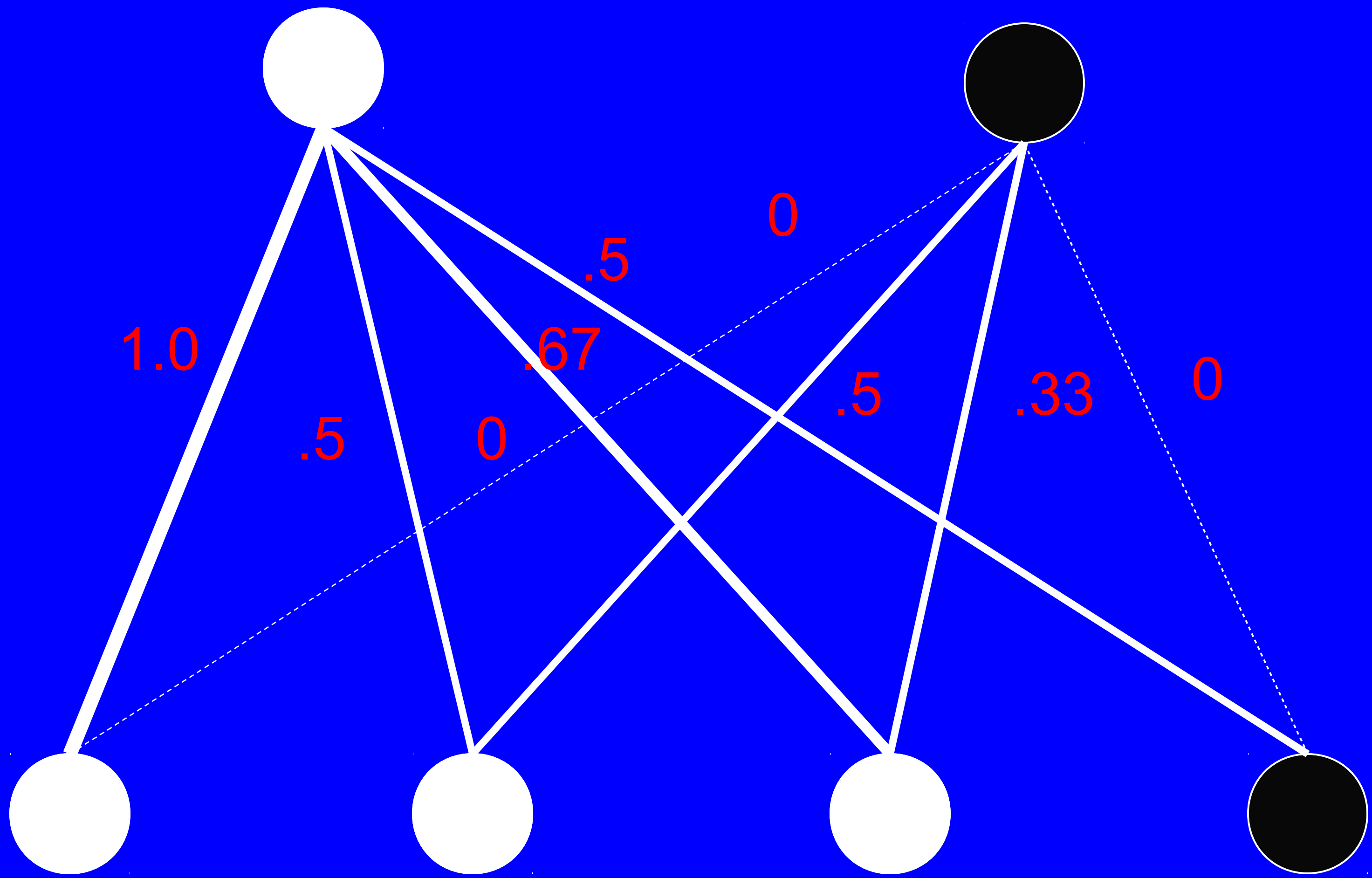


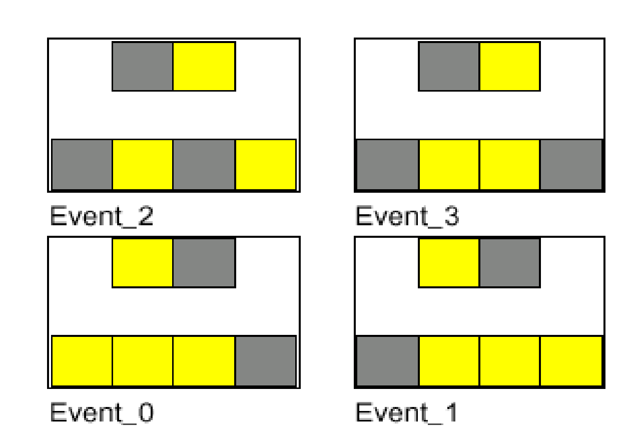
Can these weights solve the task?



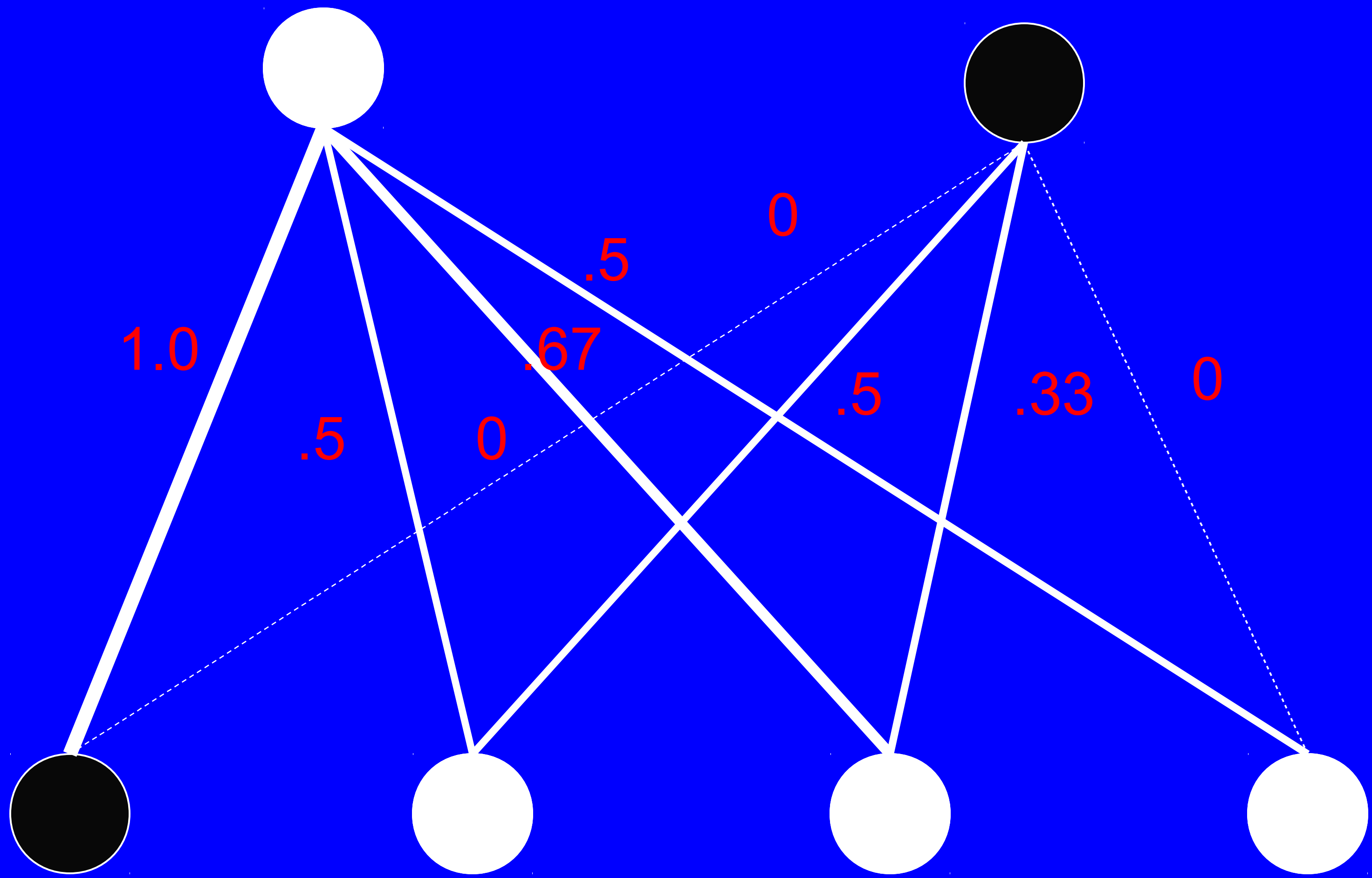


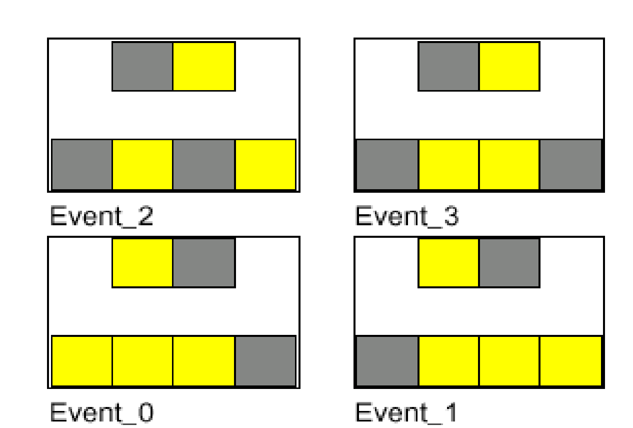
Can these weights solve the task?
Event 0 OK!



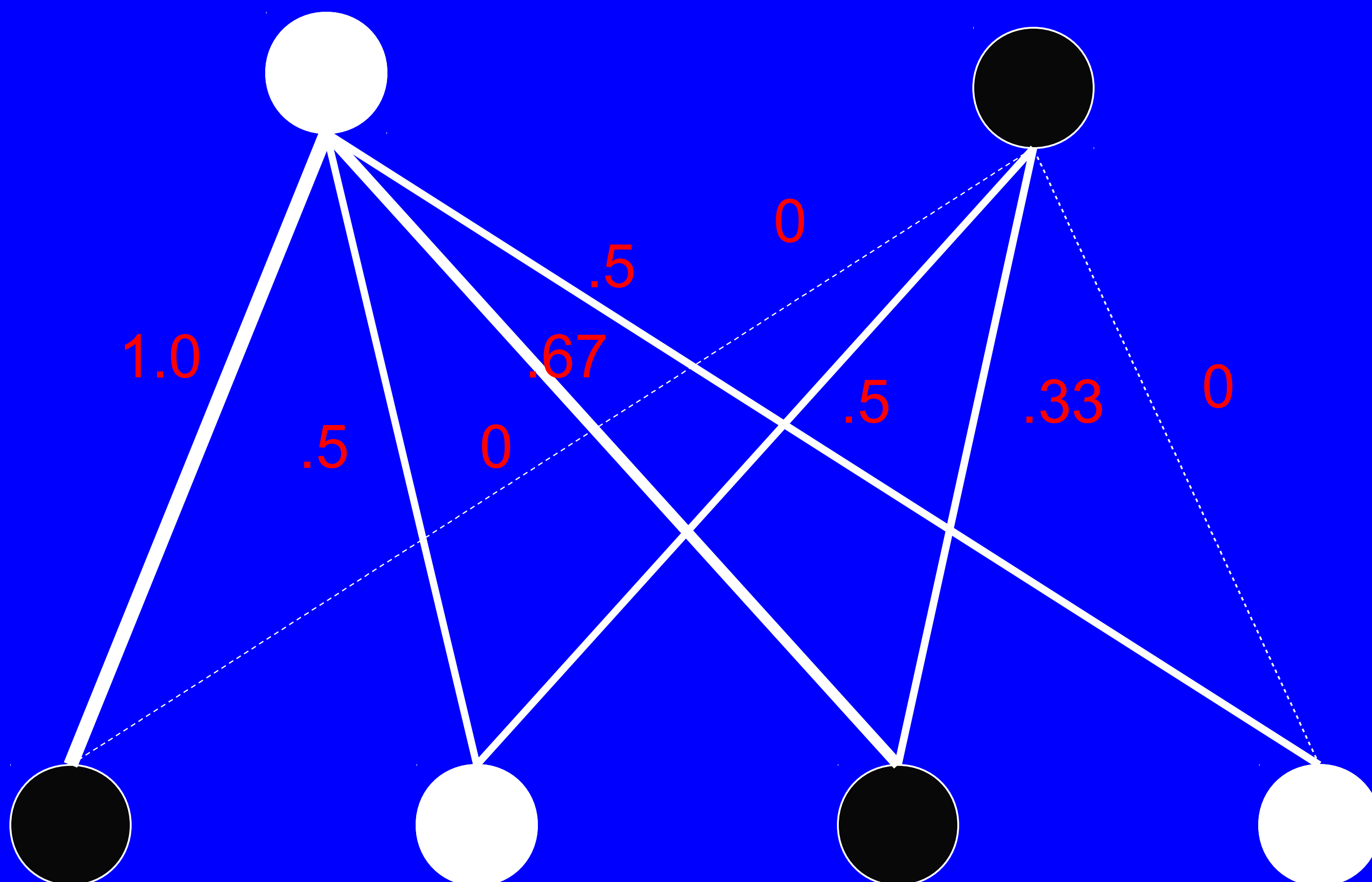


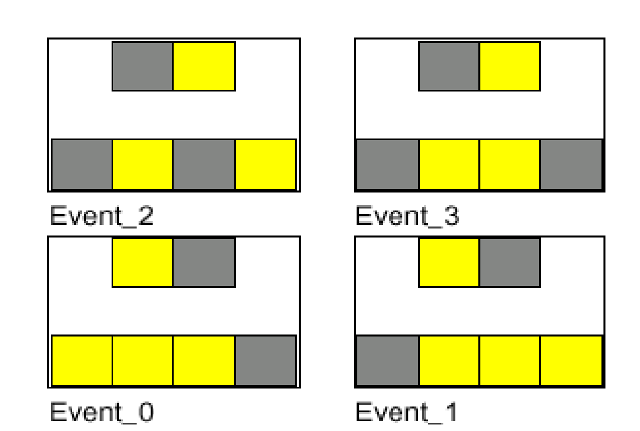
Can these weights solve the task?
Event 1 OK!



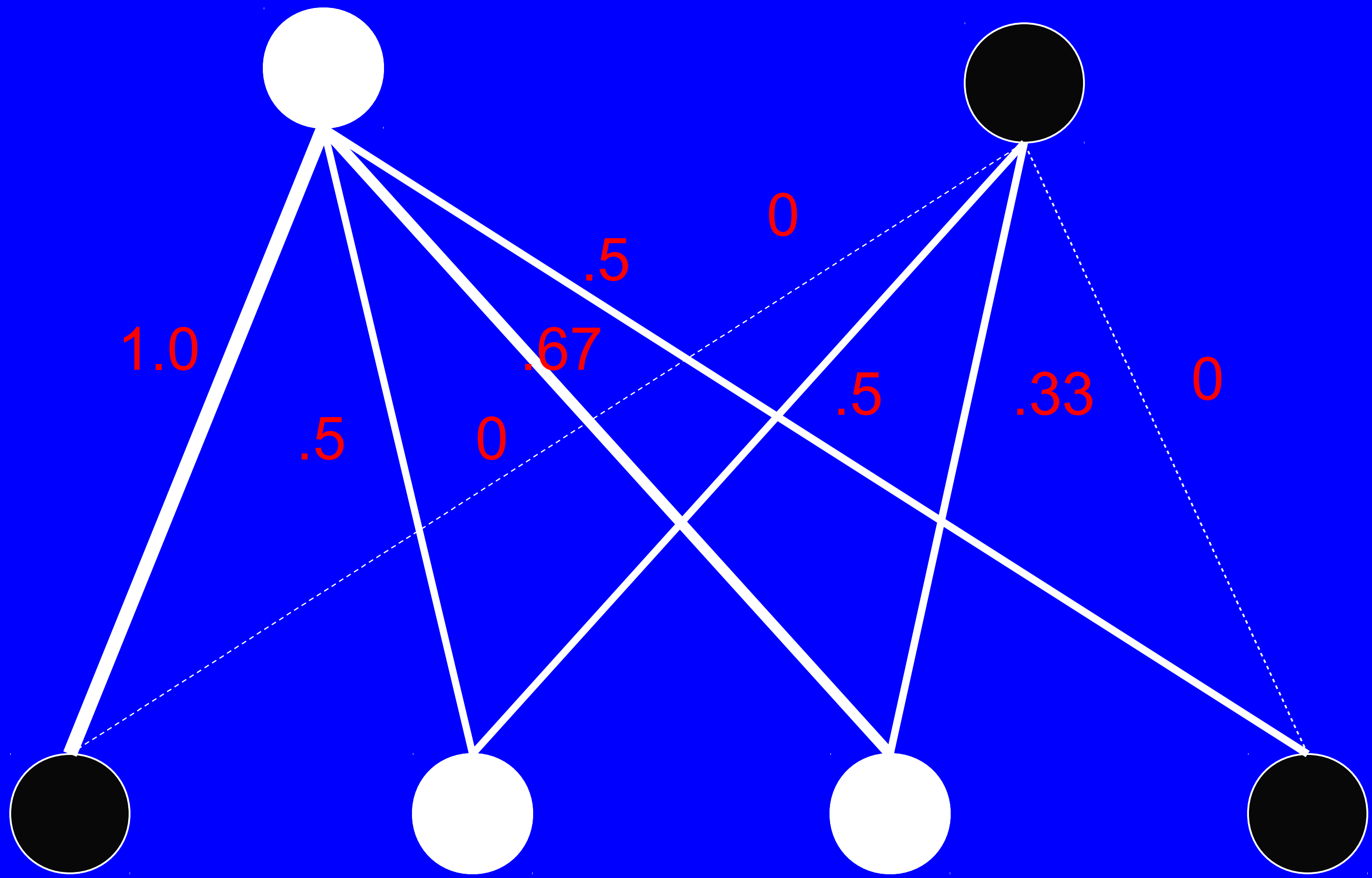


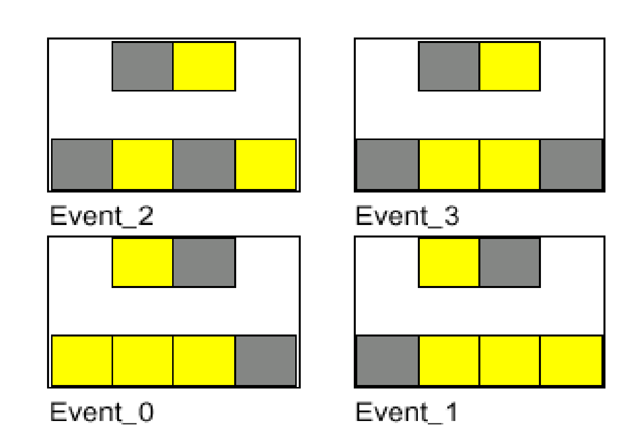
Can these weights solve the task?
Event 2 not OK....



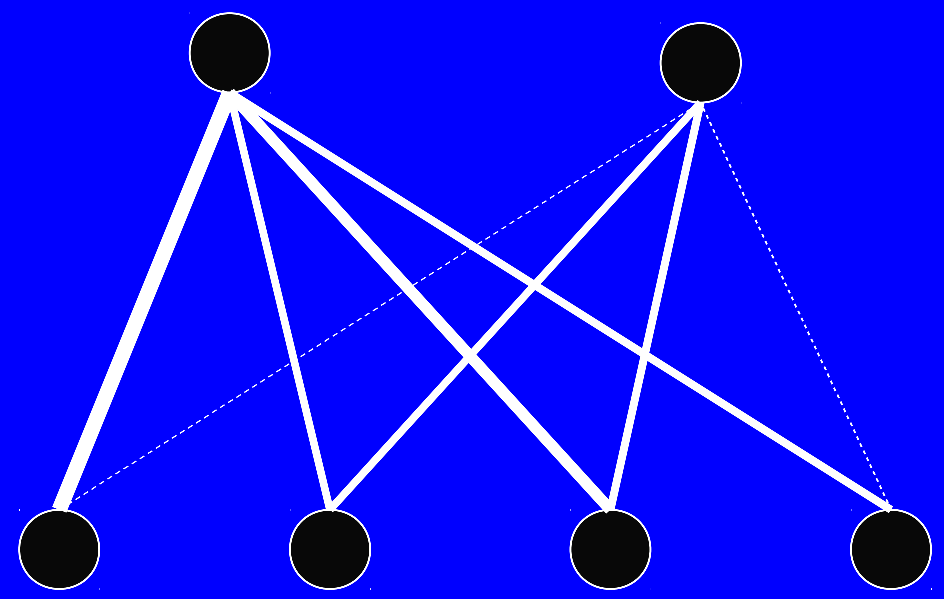


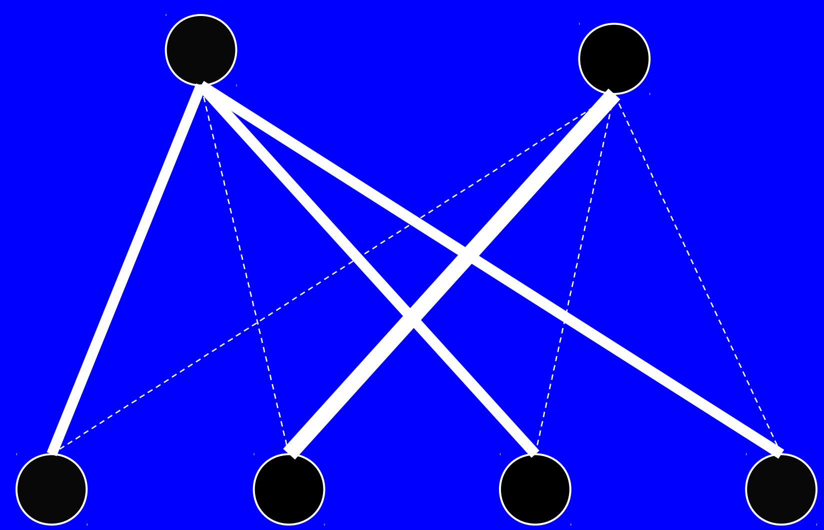
Can these weights solve the task?
Event 3 not OK....





Weights learned by Hebb =>





<= (one set of)
Weights that solve the task

Solution: Error-Driven Learning

First, we will consider how to do this and later come back to biology and more realistic implementation

- Instead of learning based on correlations, learn based on **error**: The difference between what the network is **supposed** to do, and what it **actually does**

- Error can be indexed using sum squared error (SSE)

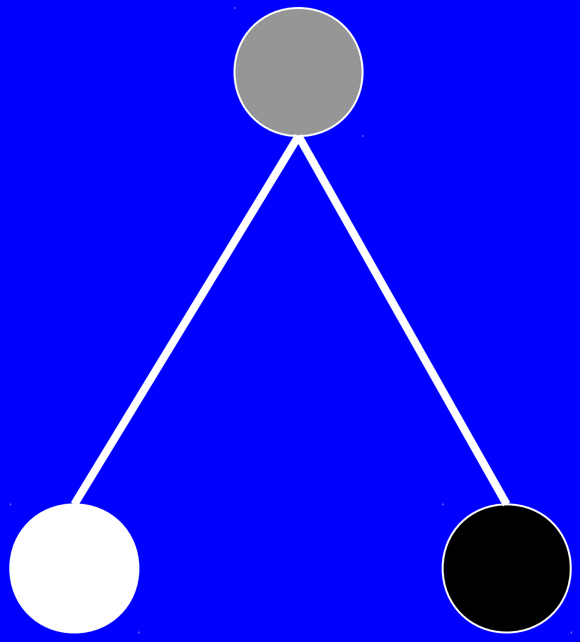
$$SSE = \sum_{p} \sum_{k} \left(t_{kp} - o_{kp}\right)^2$$

- *t* = target output value (what activation is supposed to be) over all output units *k*, summed across all input patterns *p*
- *o* = actual output values for each *k* unit and input pattern *p*

Adjusting Weights to Minimize Error

- Say that we want hidden activity = 1 for this input pattern.

- If you could pick one (of the two) weights to increment, which would you change?



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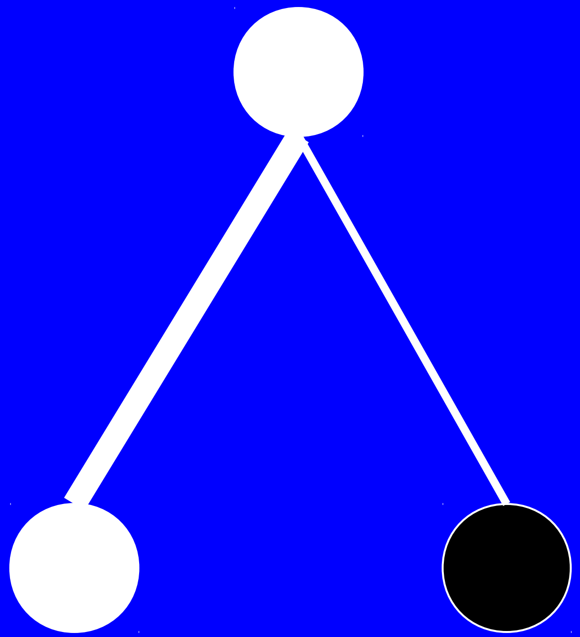
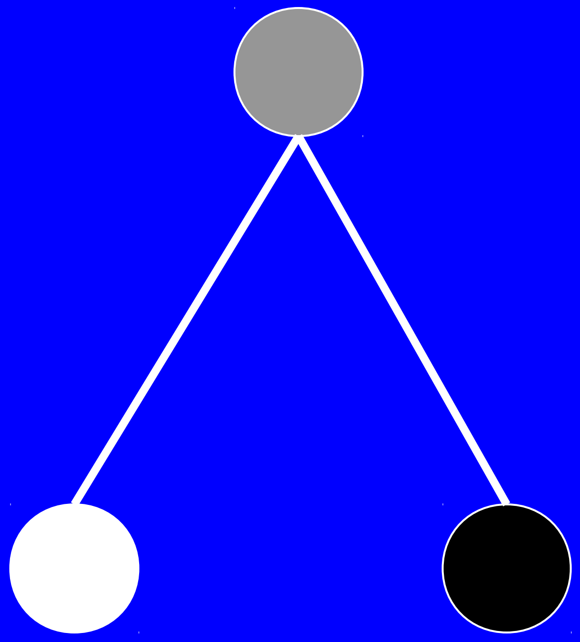


Adjusting Weights to Minimize Error

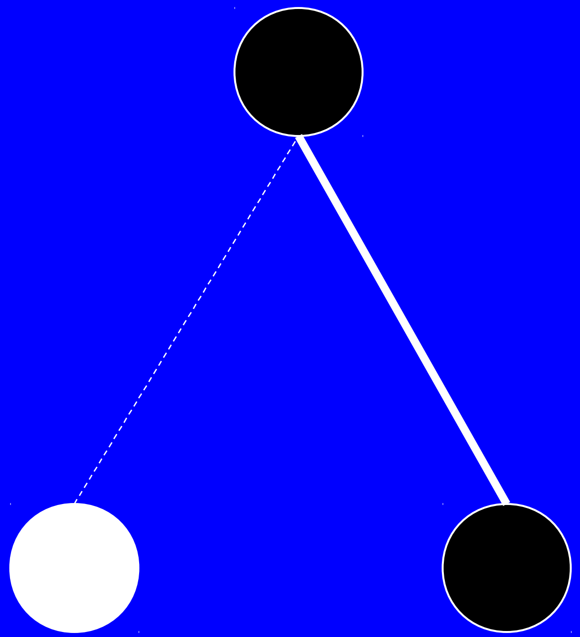
- Say that we want hidden activity = 0 for this input pattern.

- If you could pick one (of the two) weights to decrement, which would you change?



Adjusting Weights to Minimize Error

- Say that we want hidden activity = 0 for this input pattern.
- If you could pick one (of the two) weights to decrement, which would you change?



Credit/Blame Assignment

- Error-driven learning is all about figuring out who to **blame** for mistakes

- If the network makes an error, you should change weights from **active input units**

- Changing weights from inactive inputs has no effect

The Delta Rule

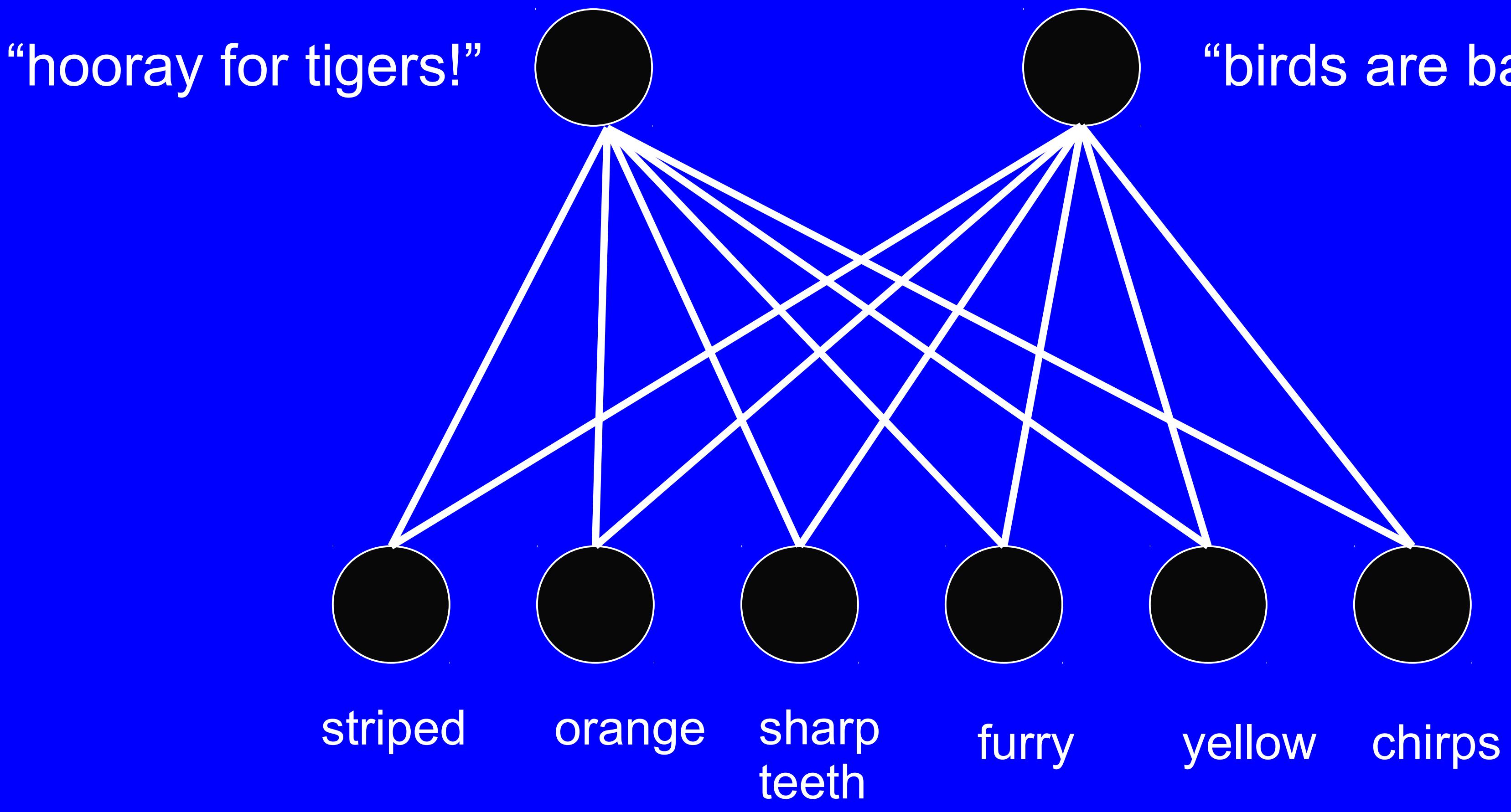
- The *delta rule* meets the criteria we have outlined for error-driven learning:

$$\Delta w_{ik} = \epsilon(t_k - o_k)s_i$$

∀ Δw_{ik} = change in weight
- t_k = target output value (what activation is supposed to be)
- o_k = actual output value
- s_i = input unit activity

- Weight change is proportional to error, and it is also proportional to sending unit activity

Error-driven learning



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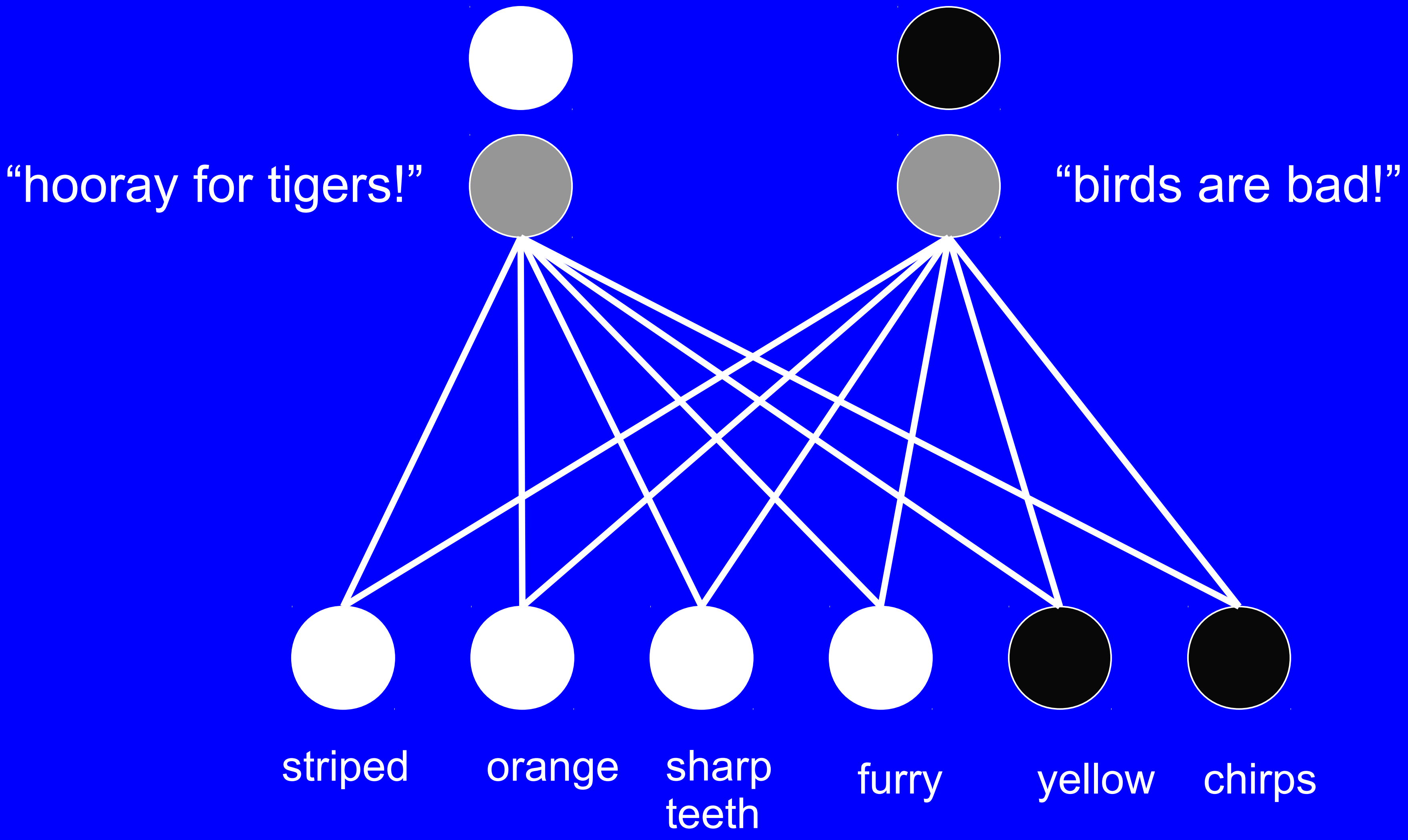

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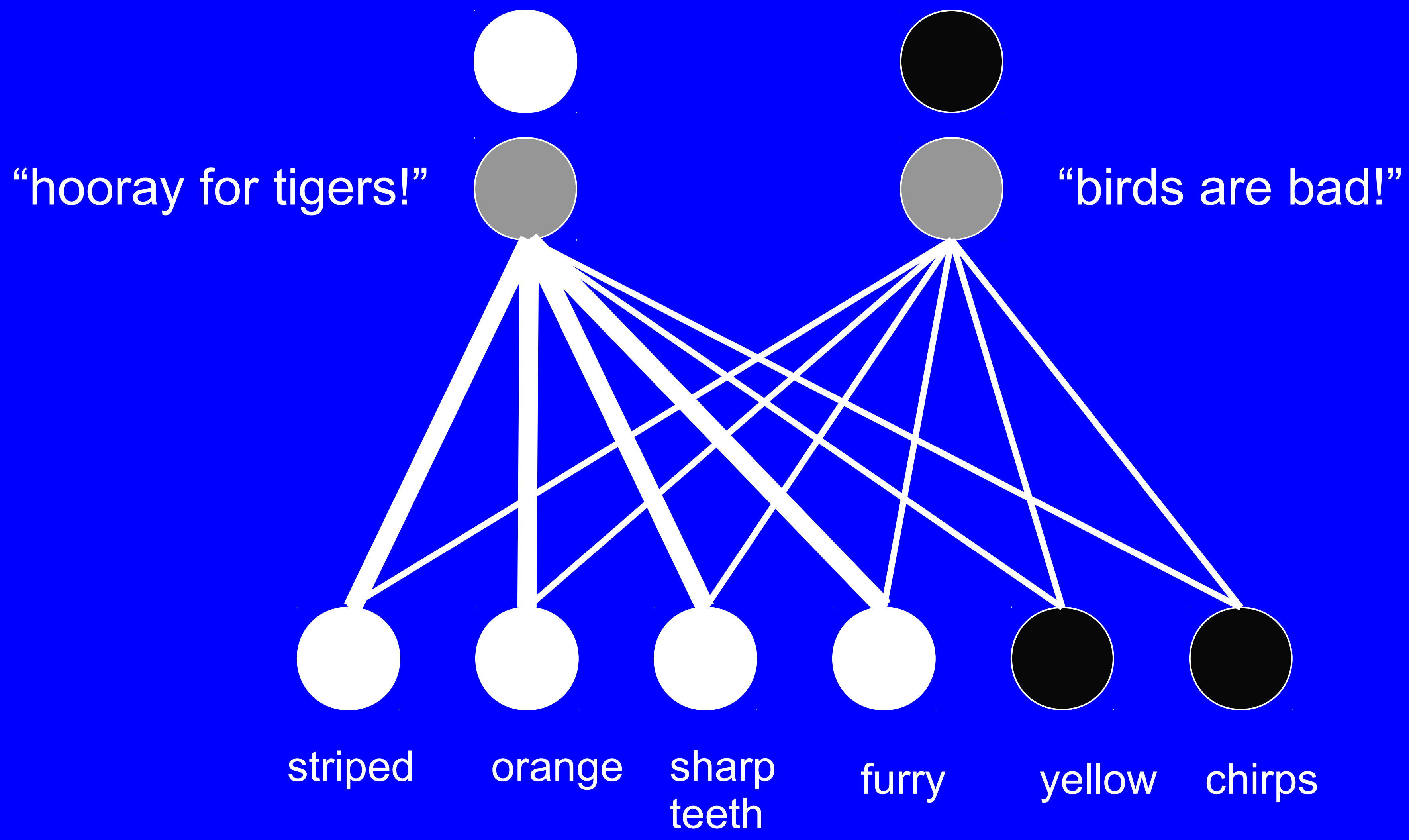

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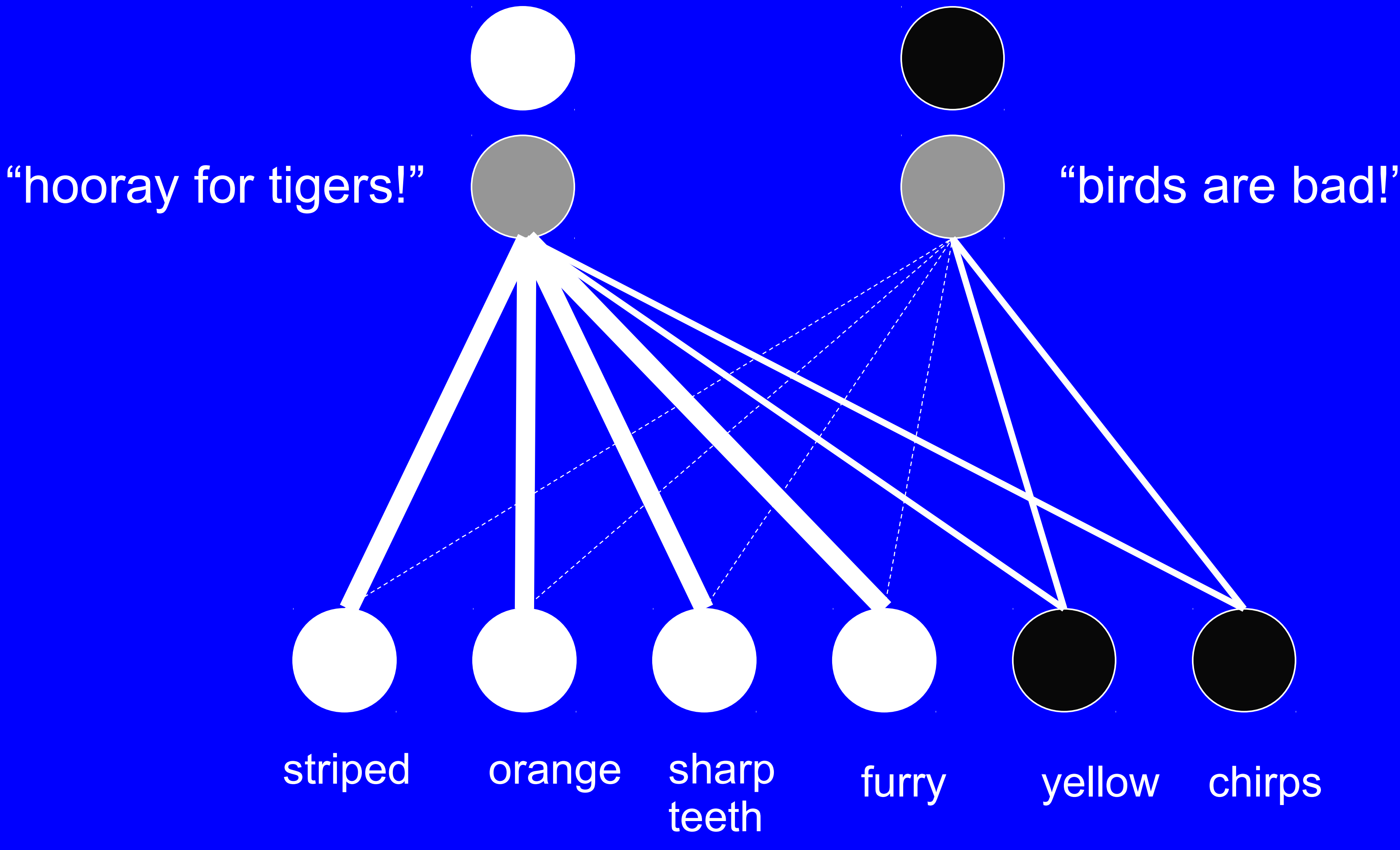

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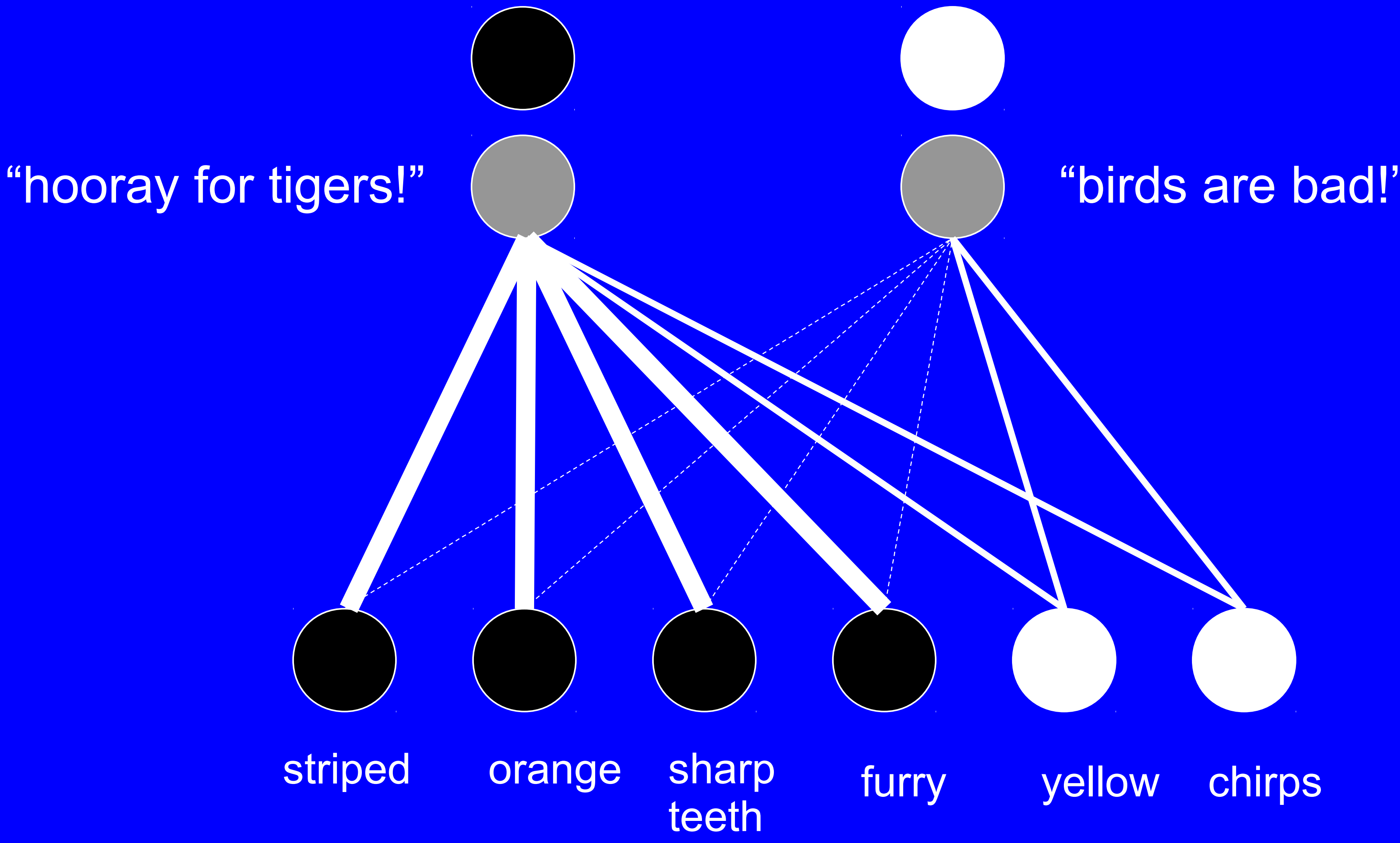

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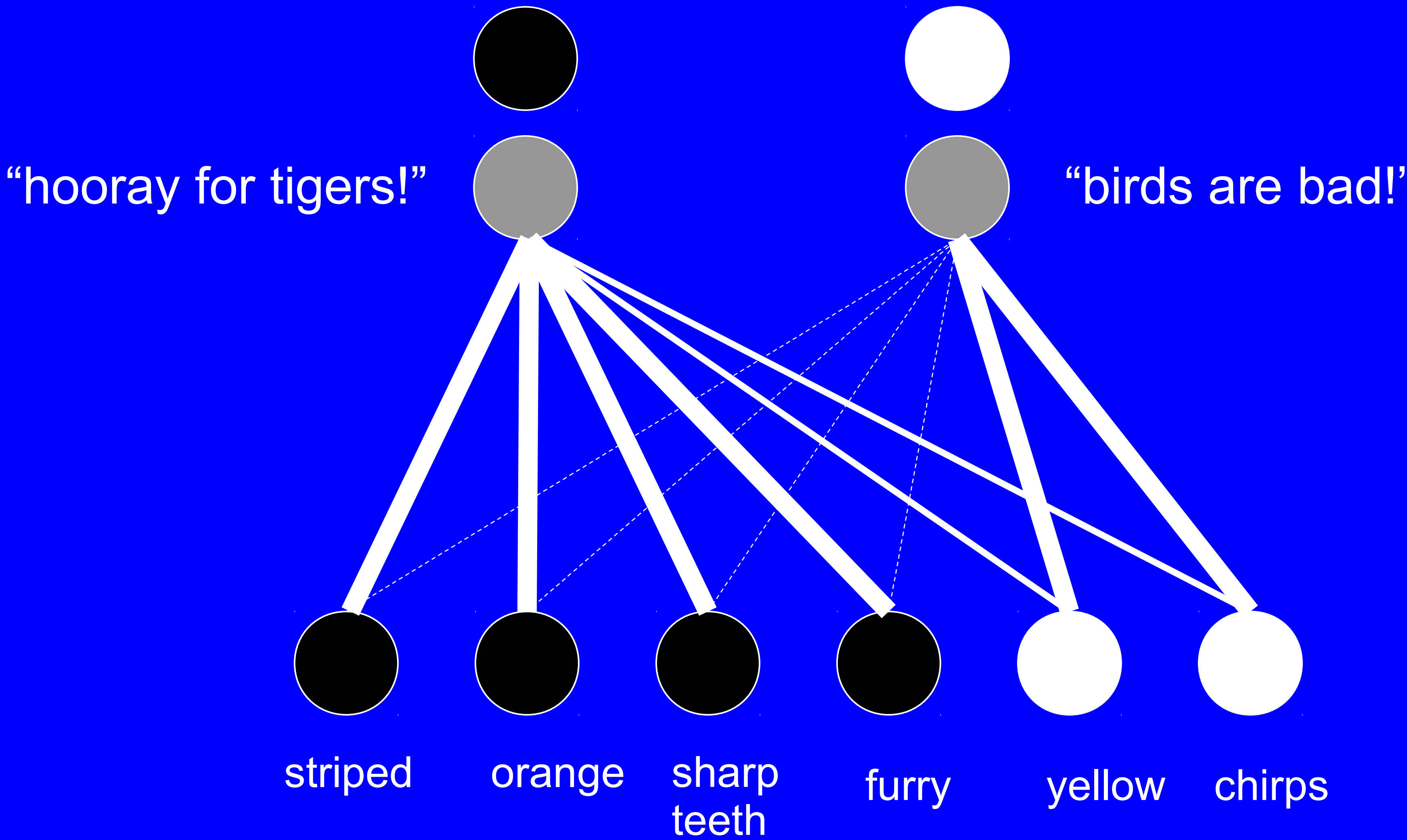

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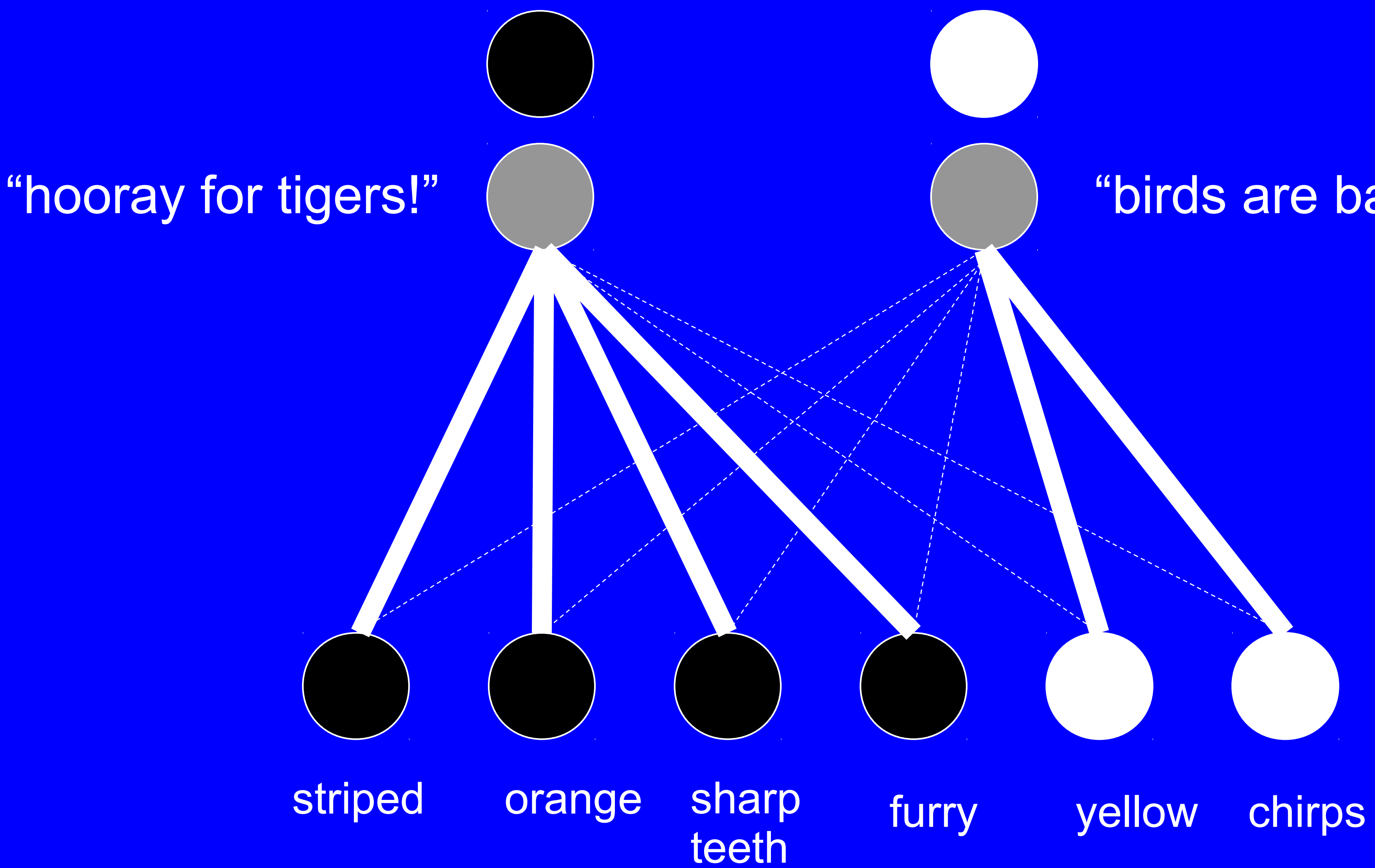
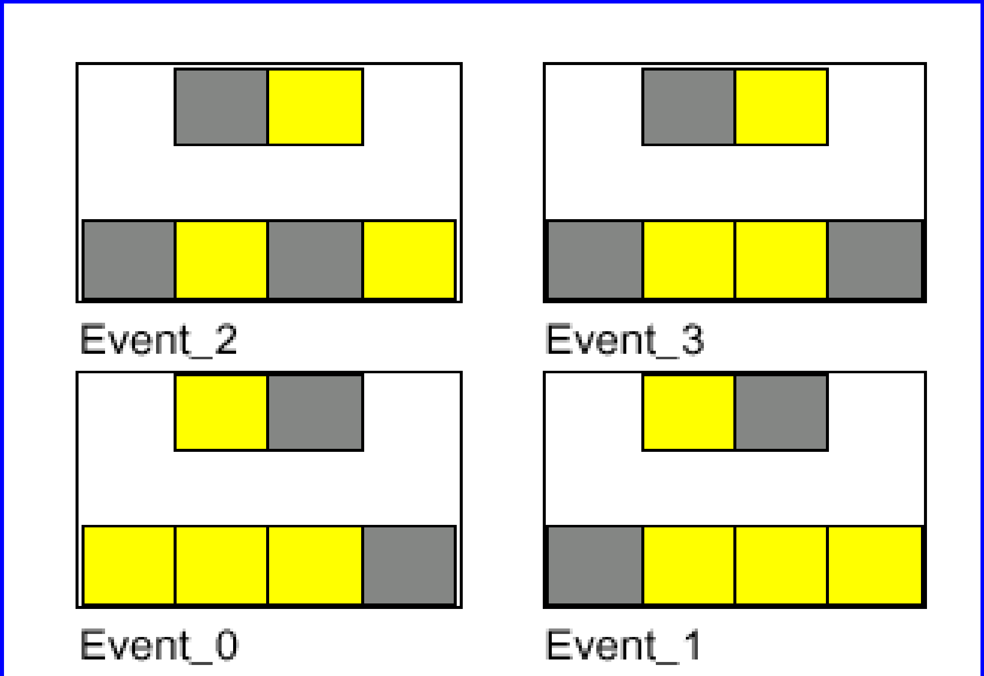
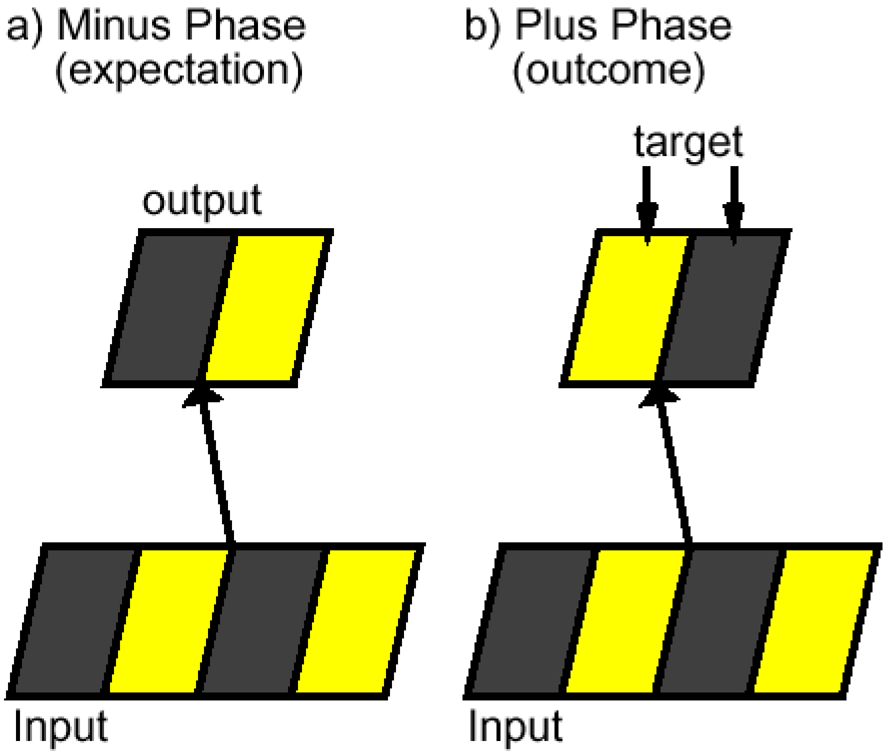


The Delta Rule and the "Hard" Problem

- The delta rule can learn the "hard" mapping that thwarted the Hebb rule



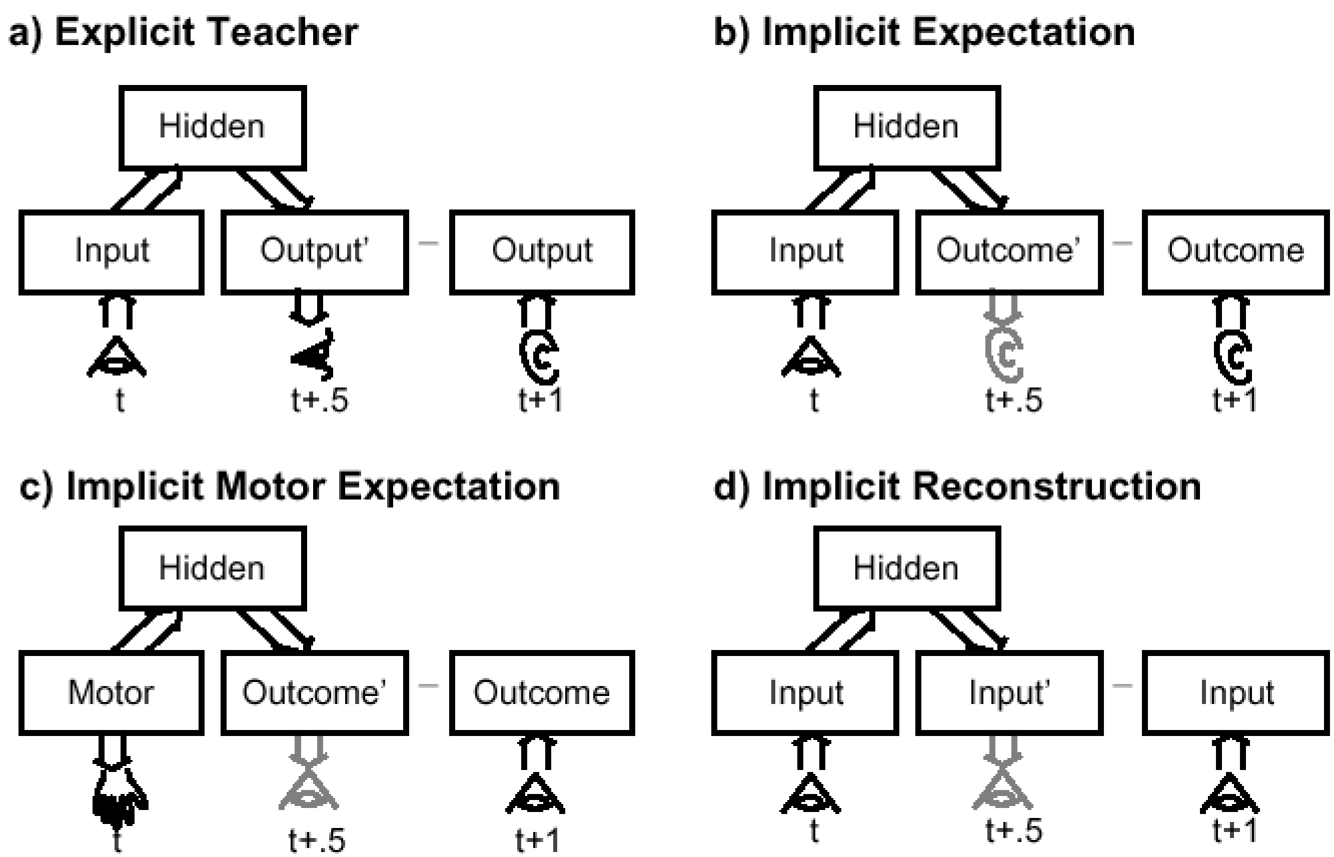
What is Target? Activation Phases



$$\Delta w_{ik} = \epsilon(o_k^+ - o_k^-)s_i$$

[pat_assoc.proj]

Nature of the Training Signals



Soft Weight Bounding

Keep weights bounded between 0-1 by exponentially slowing increases, decreases as they approach bounds:

$$\Delta w_{ik} = [\Delta_{ik}]_{+}(1 - w_{ik}) + [\Delta_{ik}]_{-} w_{ik}$$

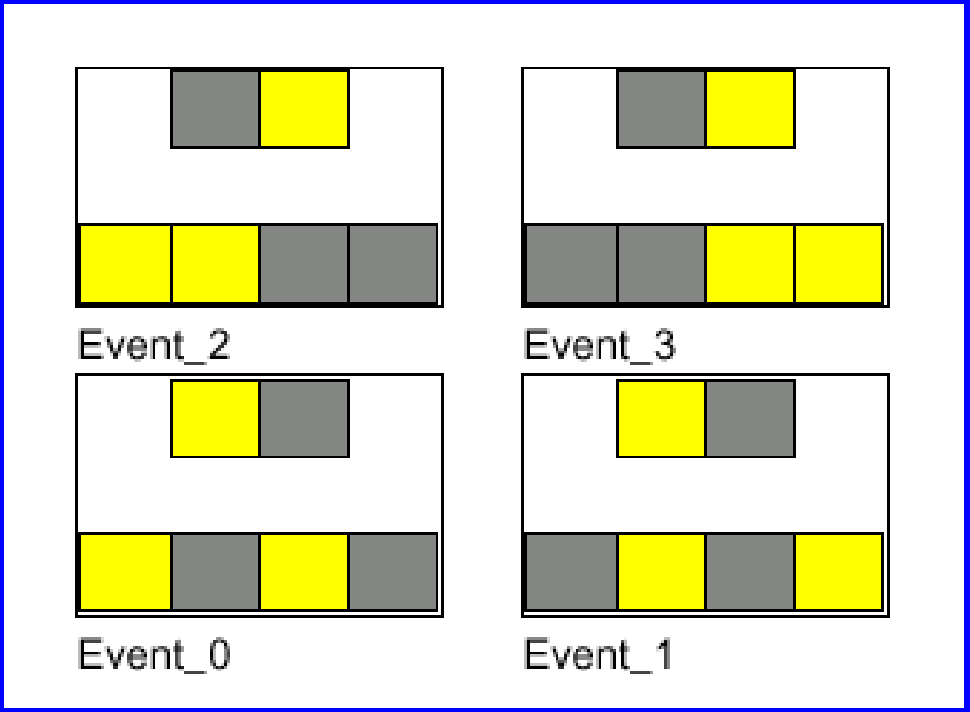
$[\Delta_{ik}]_{+}$ = computed weight change if positive (else 0).

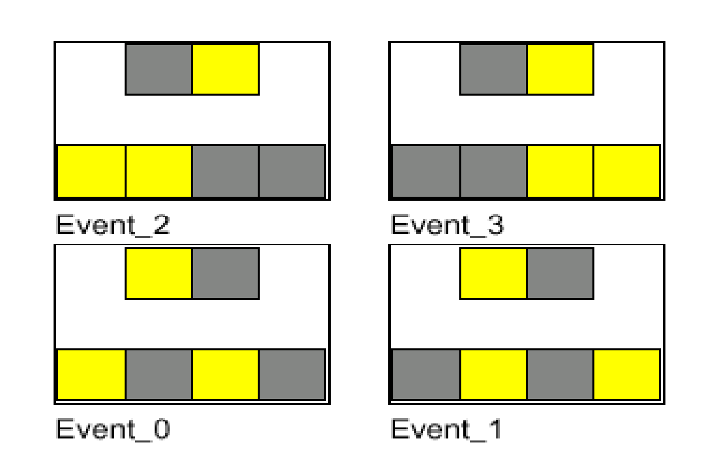
$[\Delta_{ik}]_{-}$ = computed weight change if negative (else 0).

* reflects biological constraints on number of receptors, etc. (weight can only go so high, low)

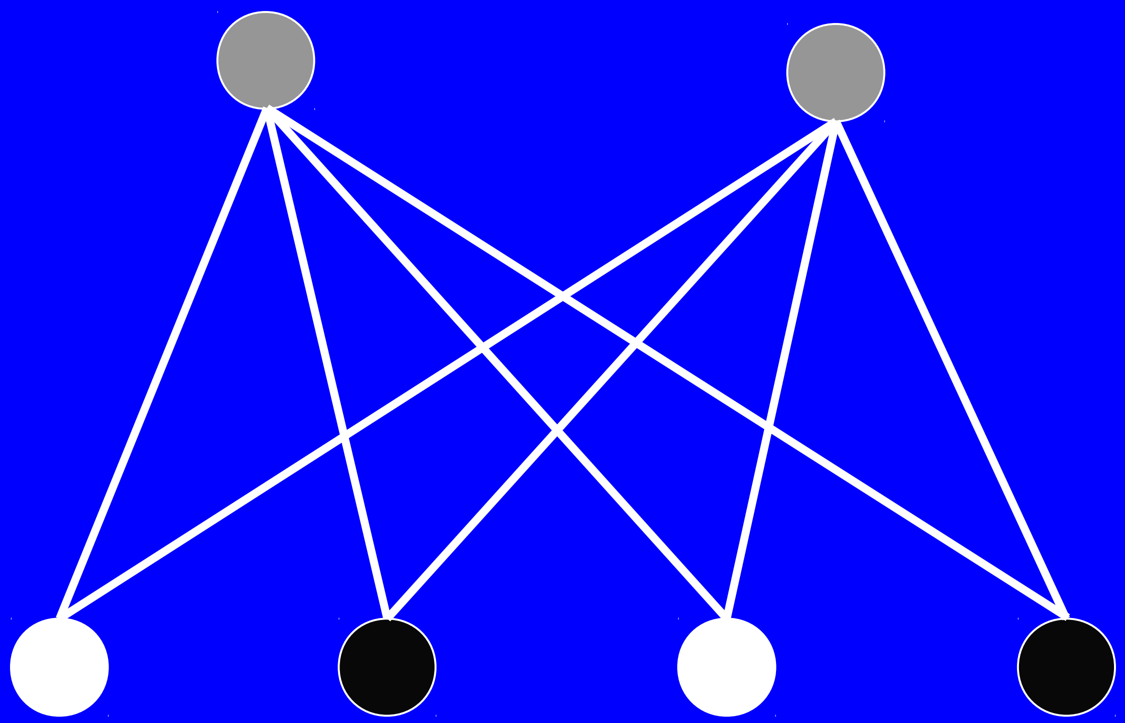
"Impossible" Mapping

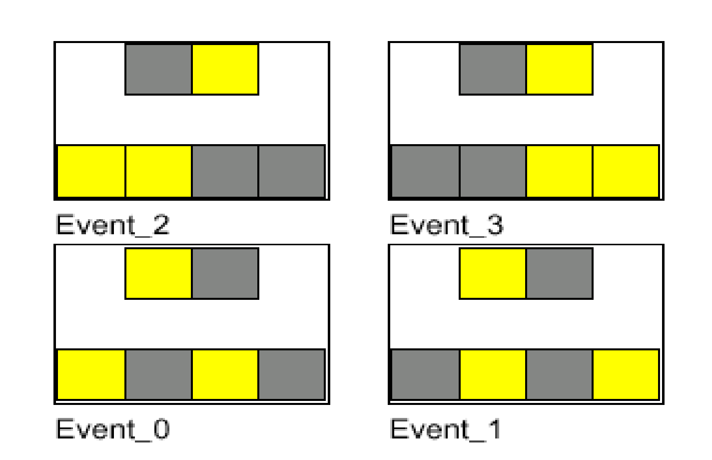
- Each input unit is linked equally often to each output unit
- Two layer networks using the delta rule can not solve this!





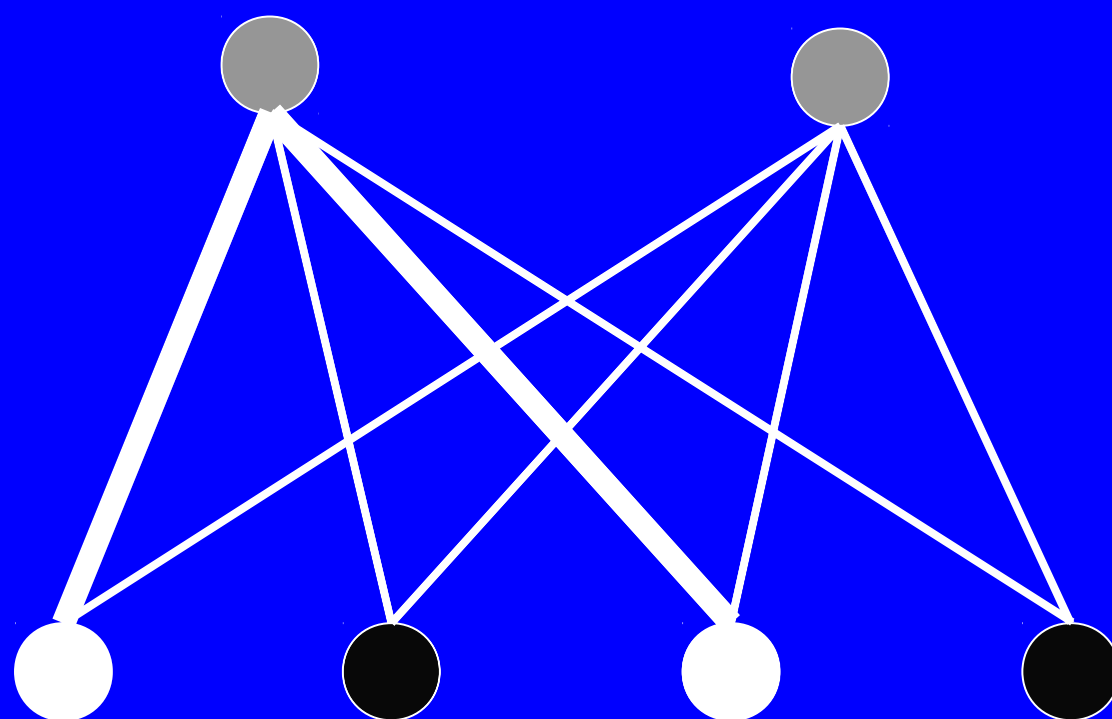
Changing weights to learn Event_0...



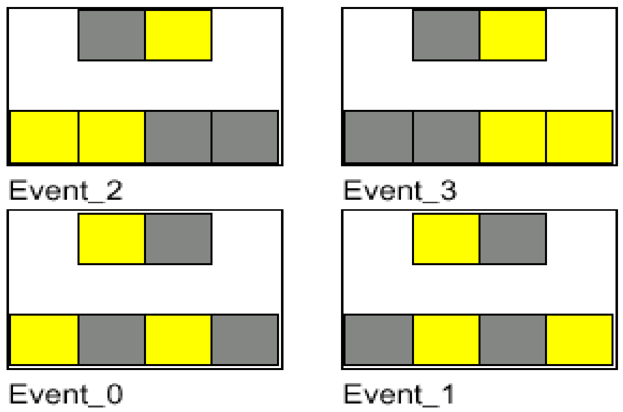


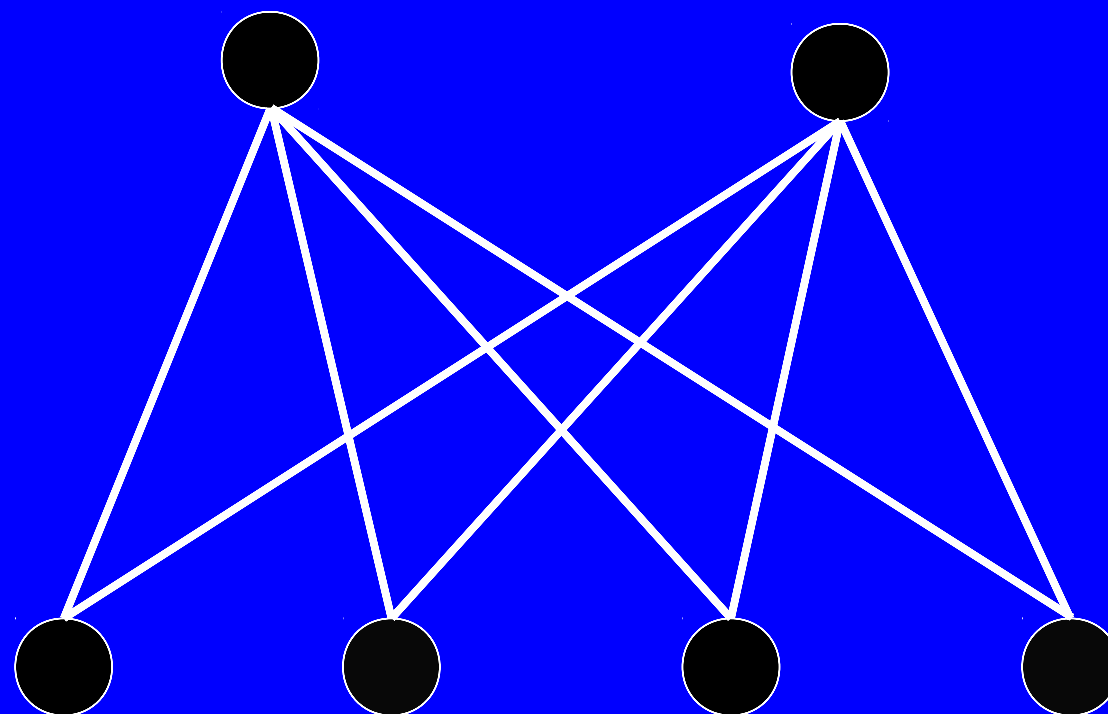
Changing weights to learn Event_0...

... hurts performance for Event_2 and Event_3

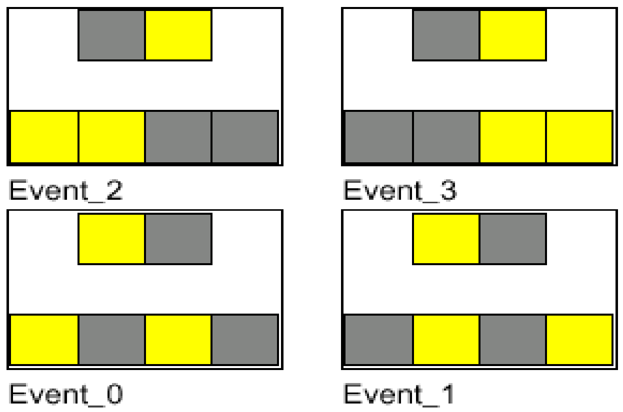


[pat_assoc.proj]

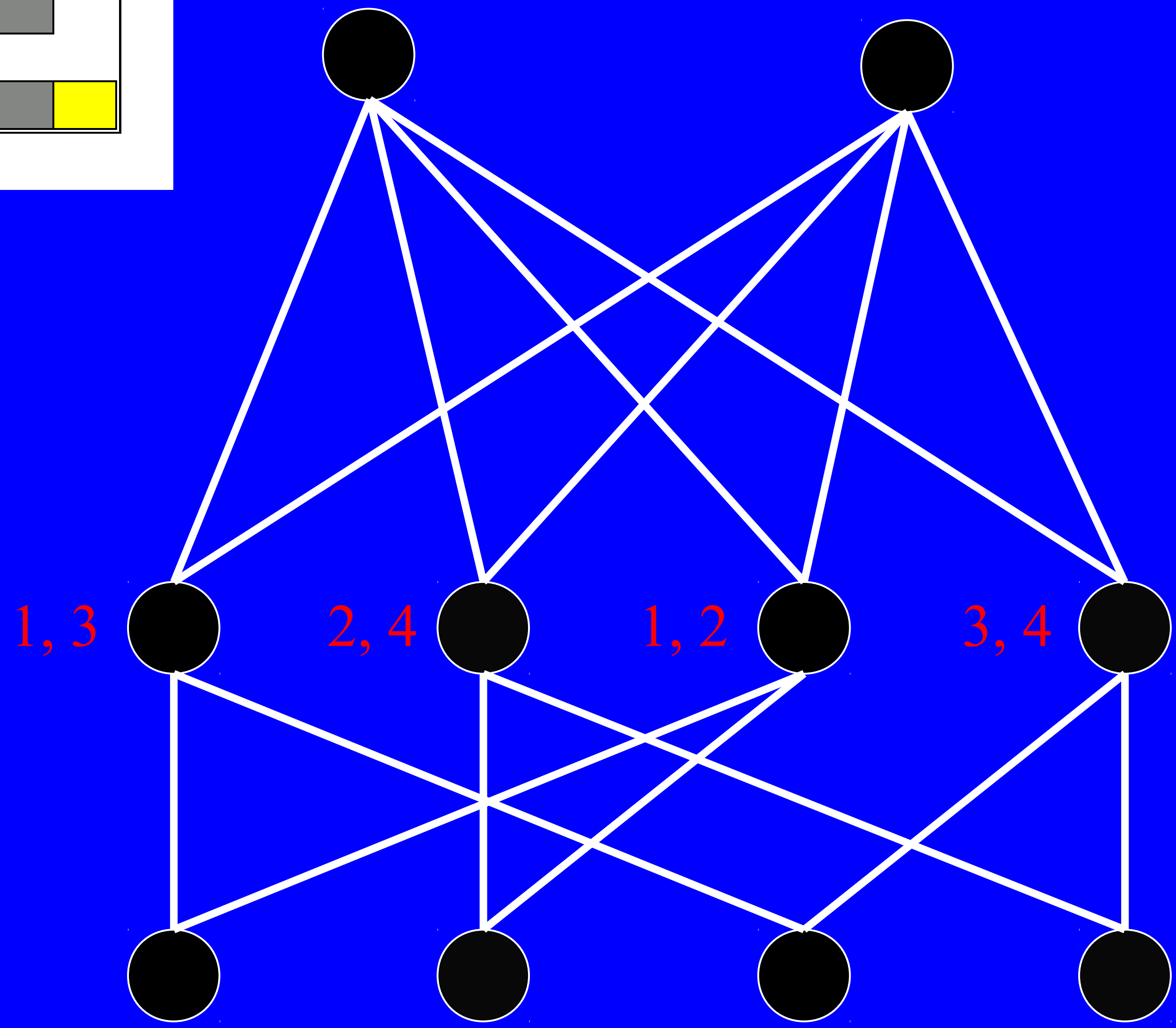

Event_2
Event_3
Event_0
Event_1

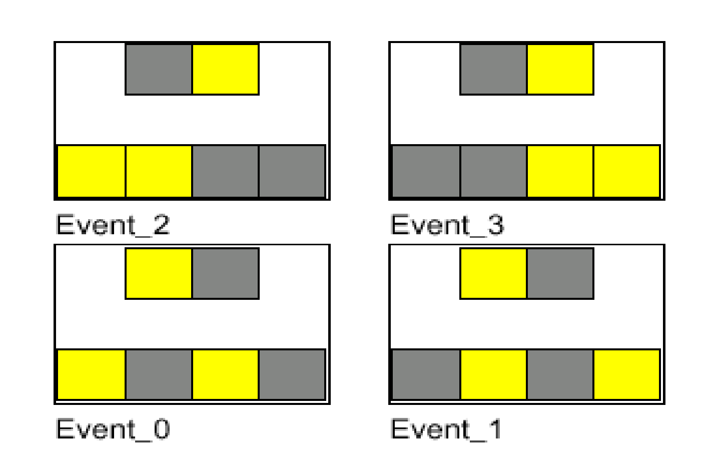


Add a hidden layer that represents feature **conjunctions** ...



hidden layer =>





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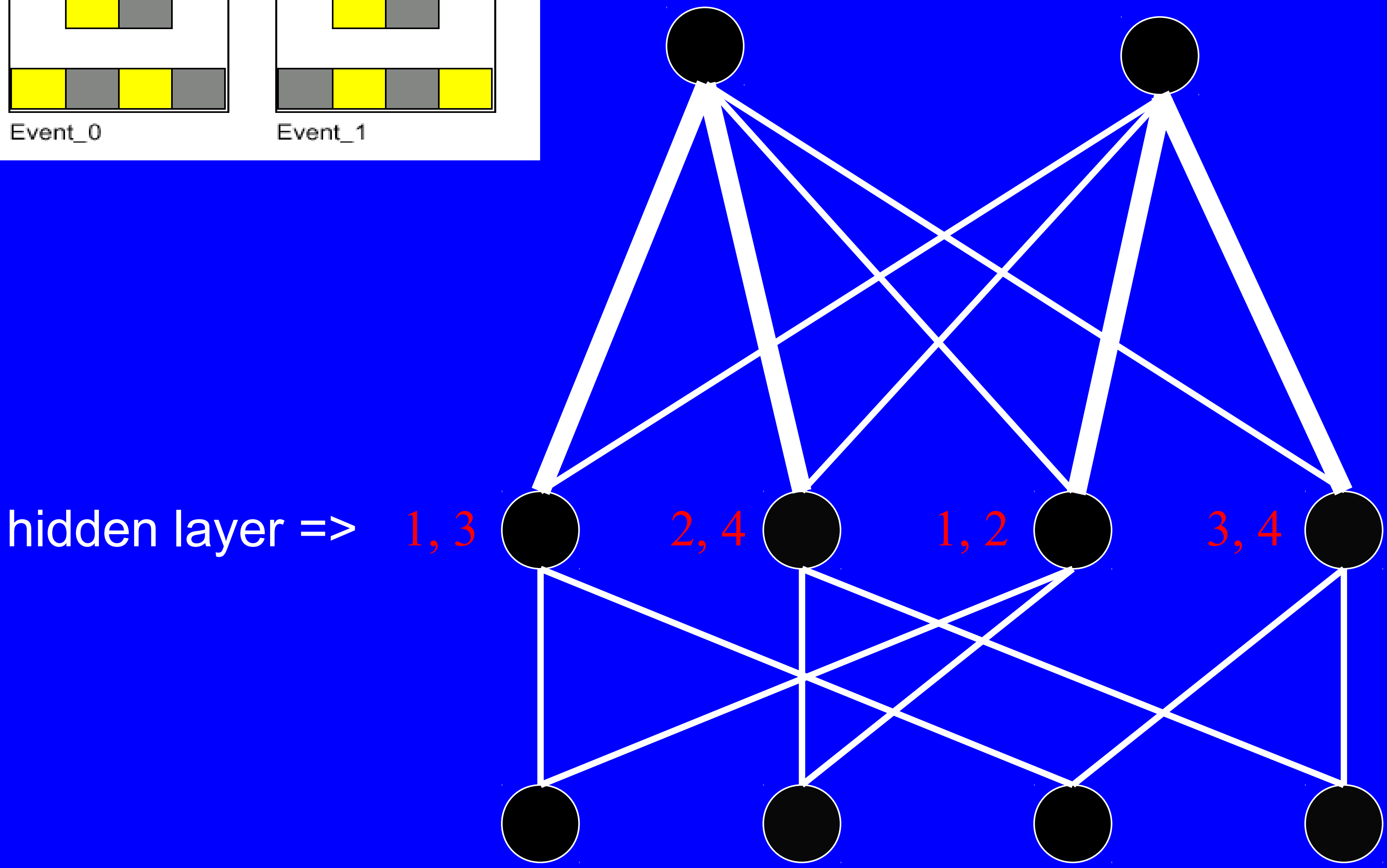

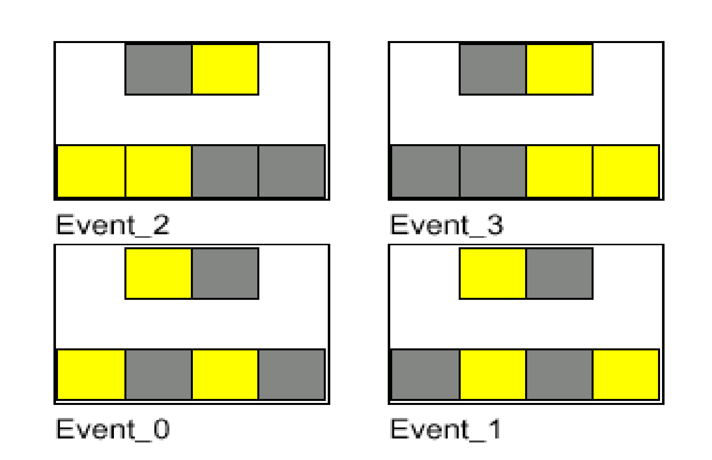

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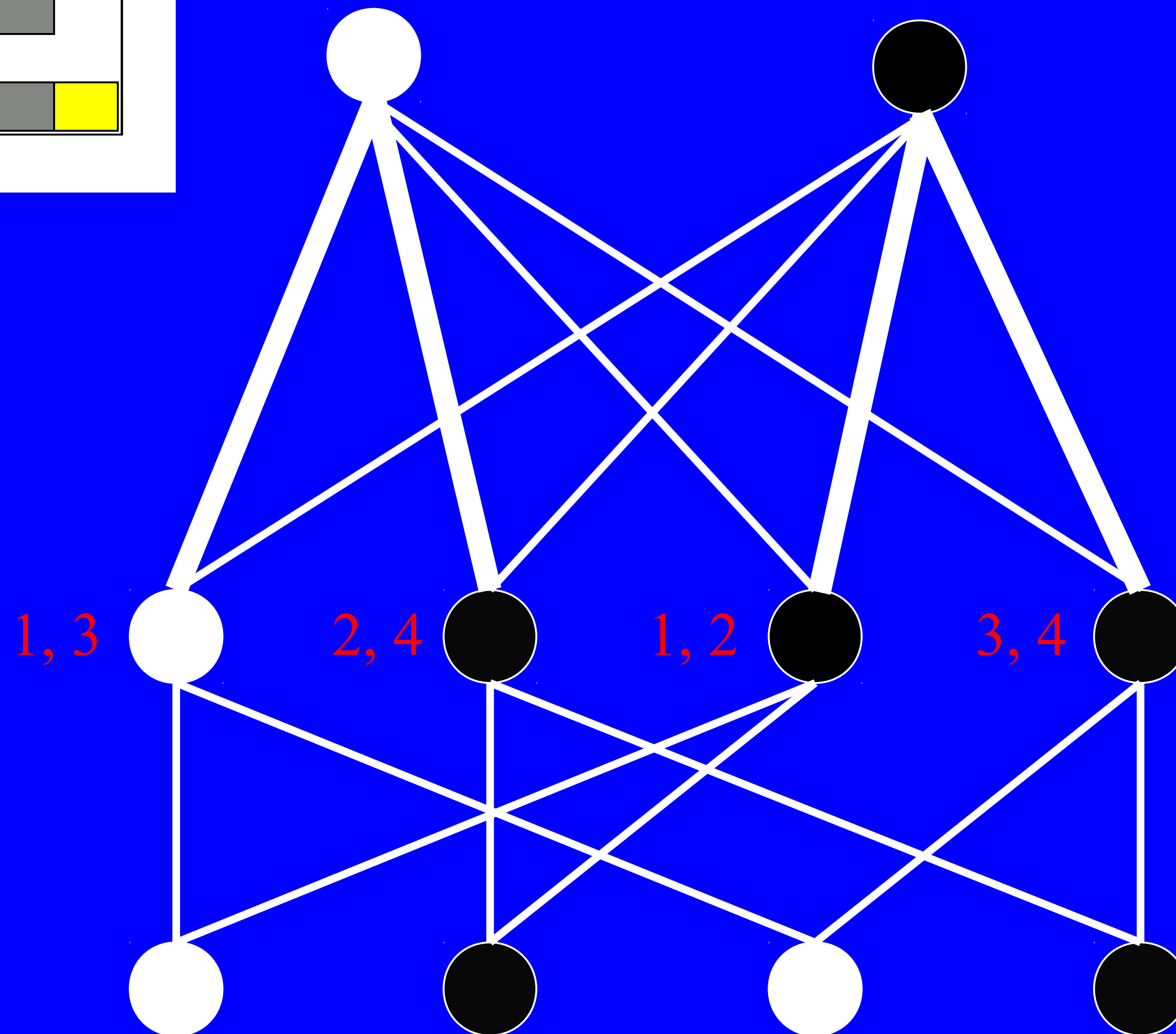

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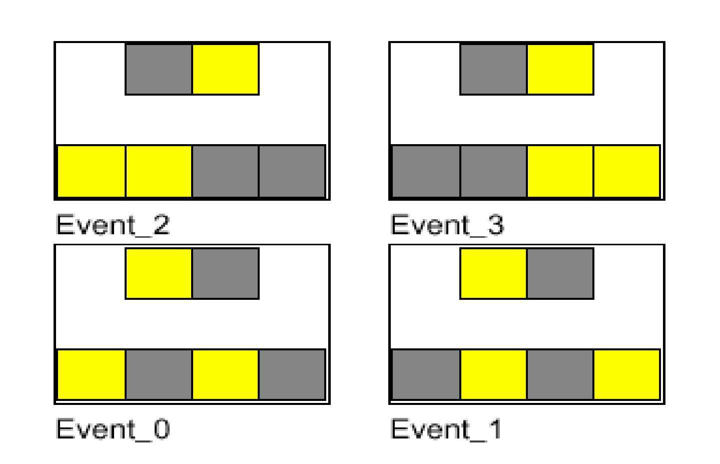

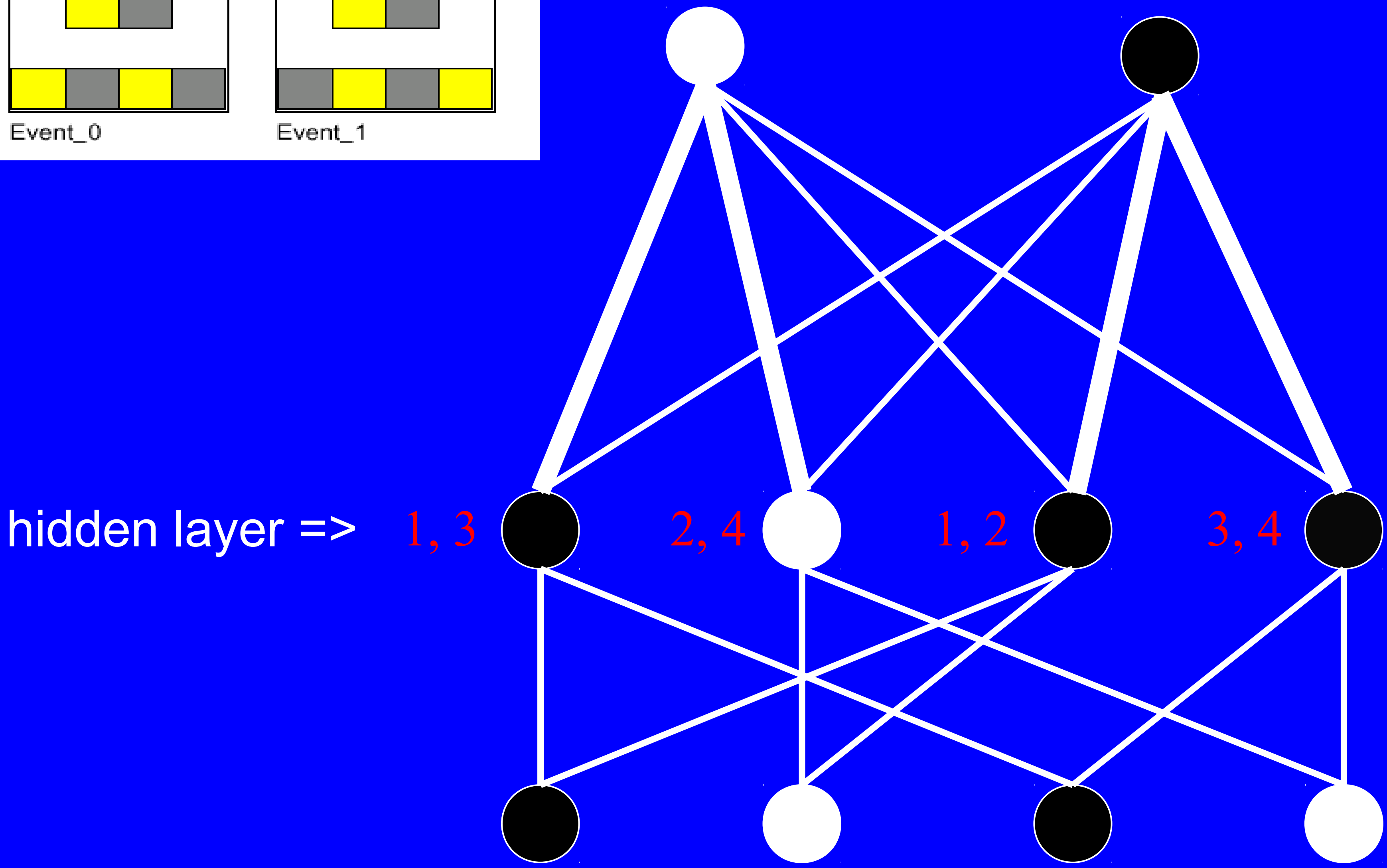

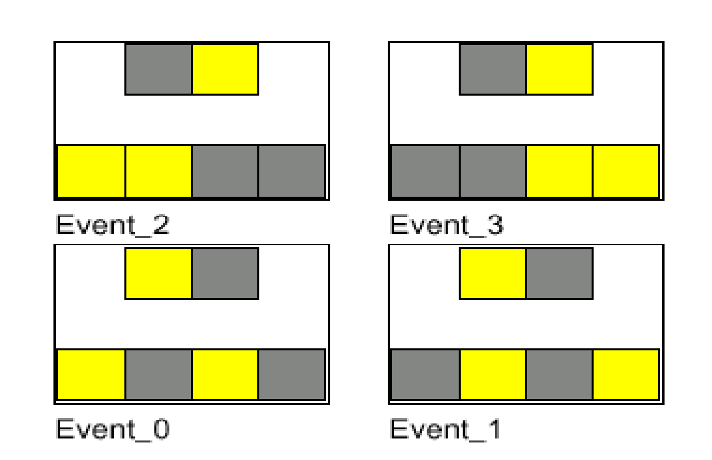

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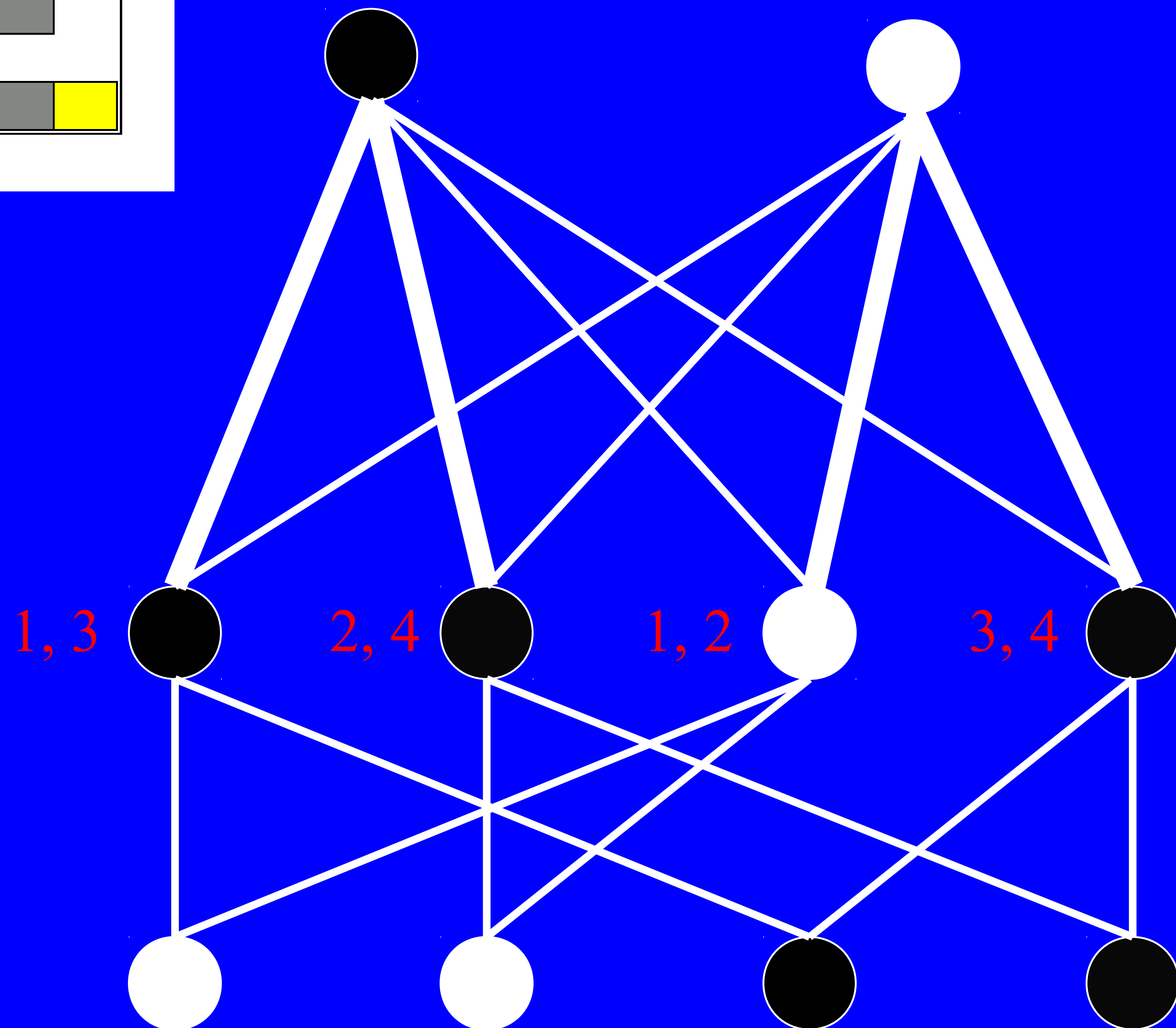

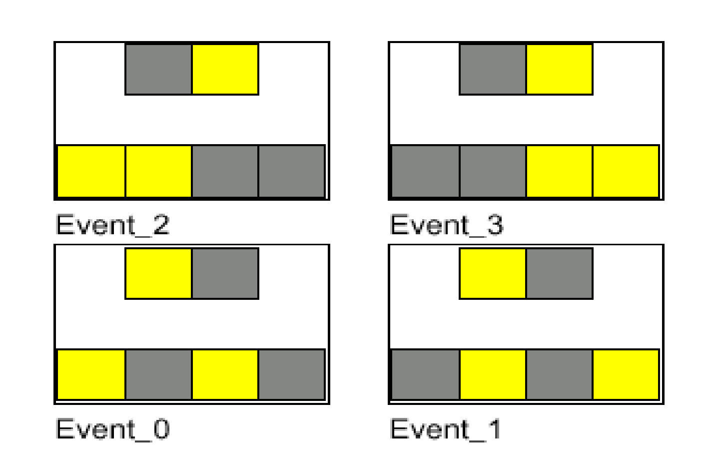

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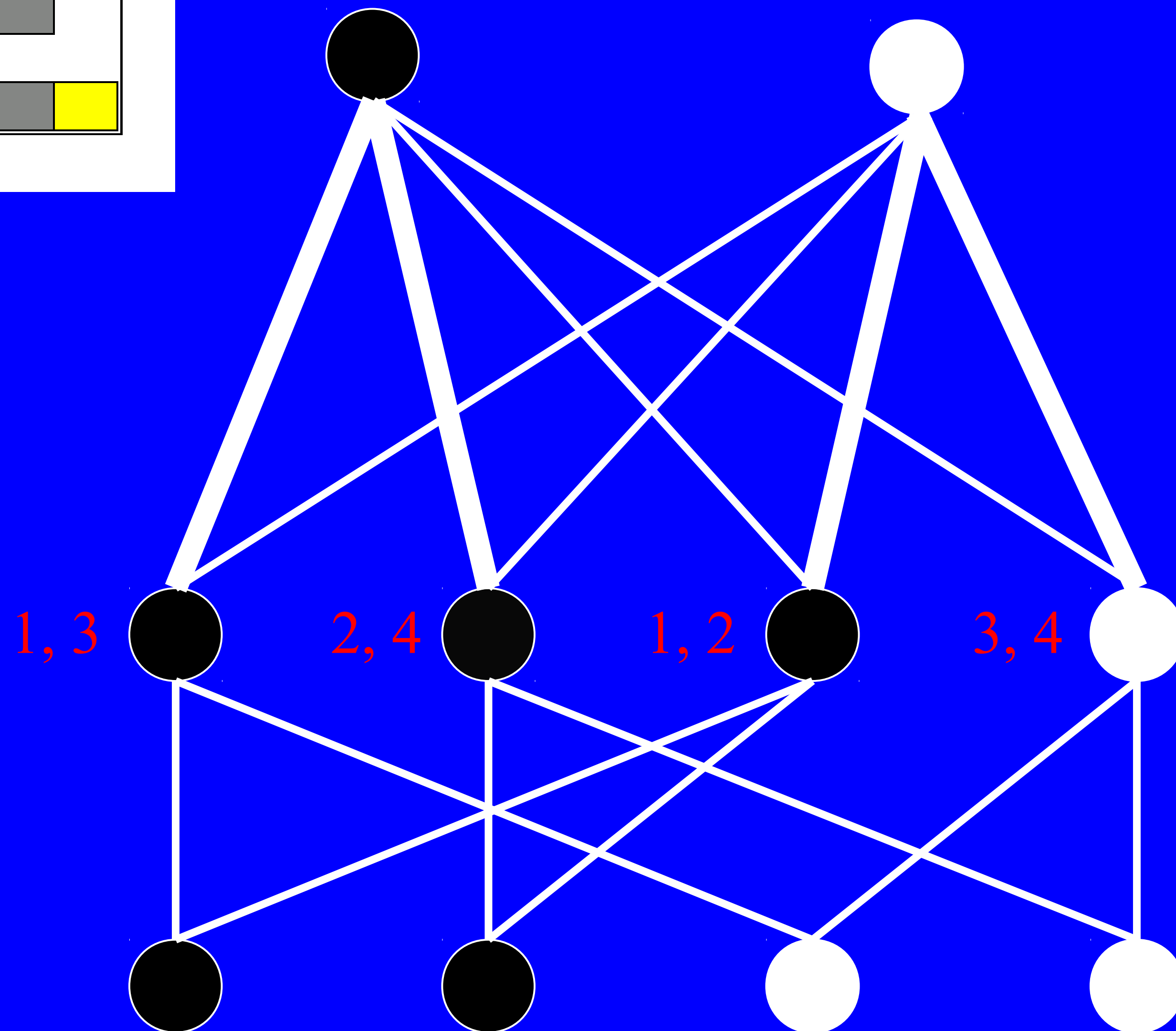
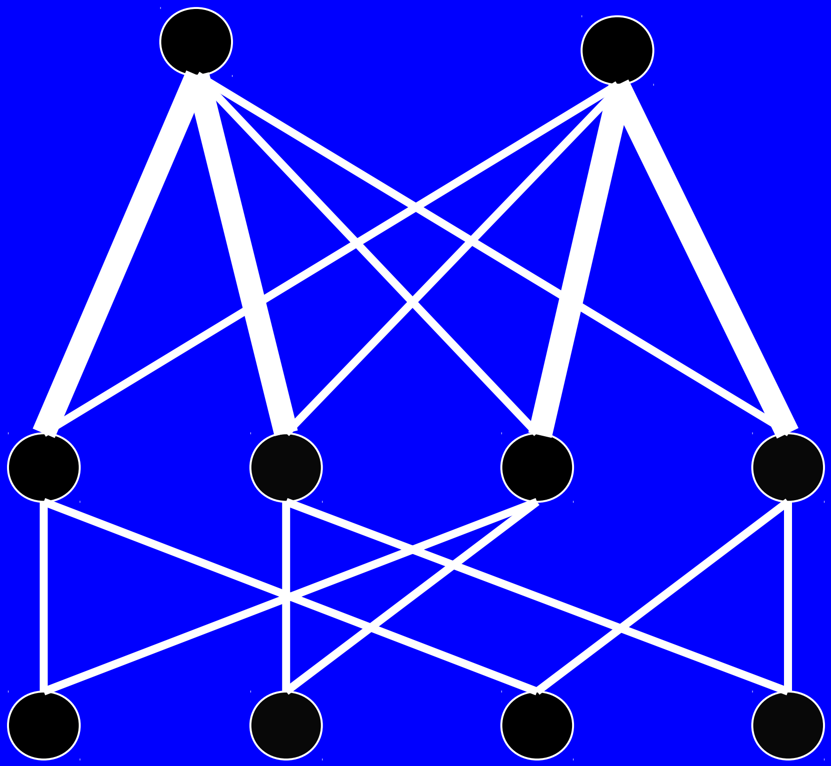
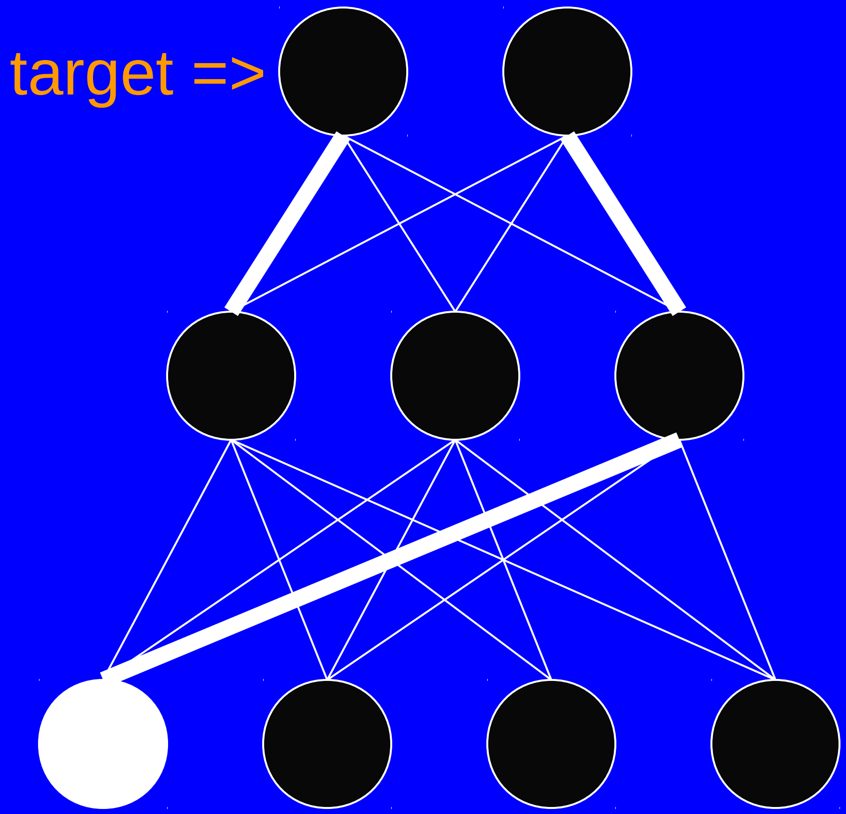


Error-Driven Learning in Multilayer Networks

- We established that networks with hidden layers can solve problems that two-layer networks can not solve, by **re-representing the input patterns**
- How do we train multi-layer networks?



Learning in Multilayer Networks



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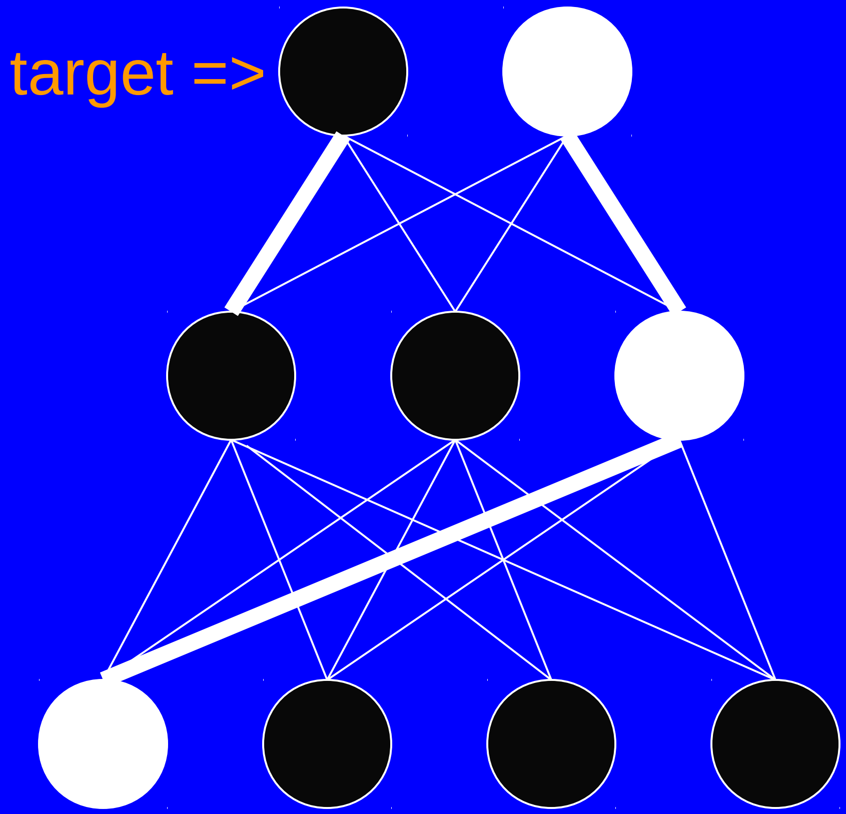

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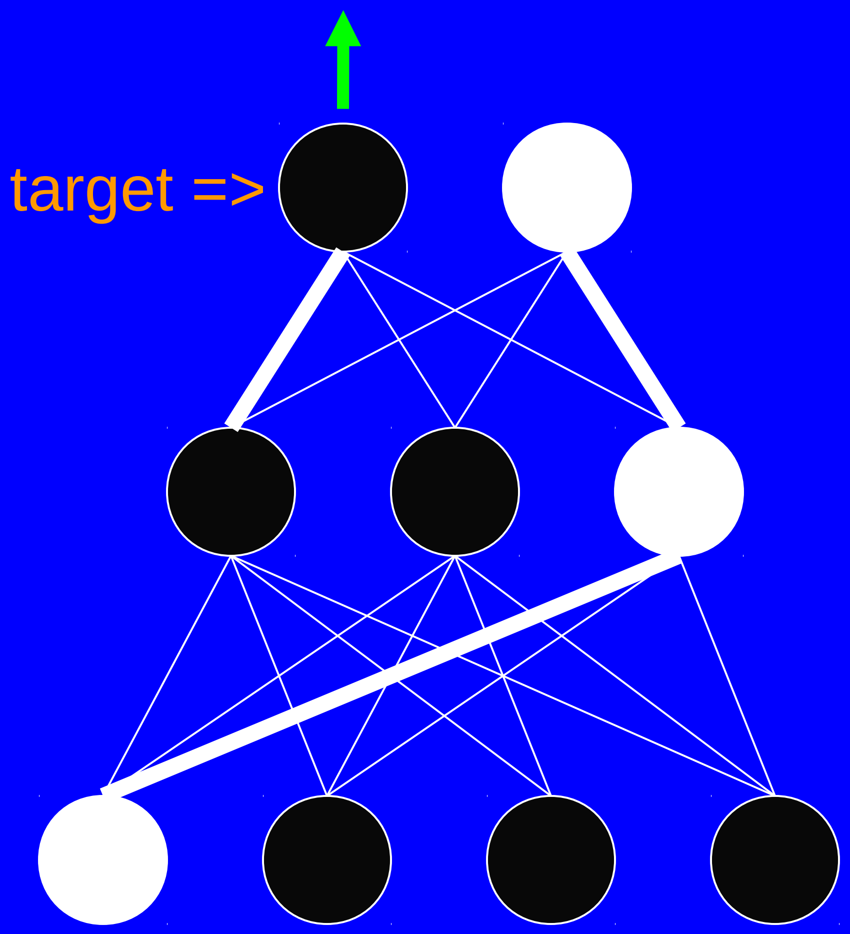

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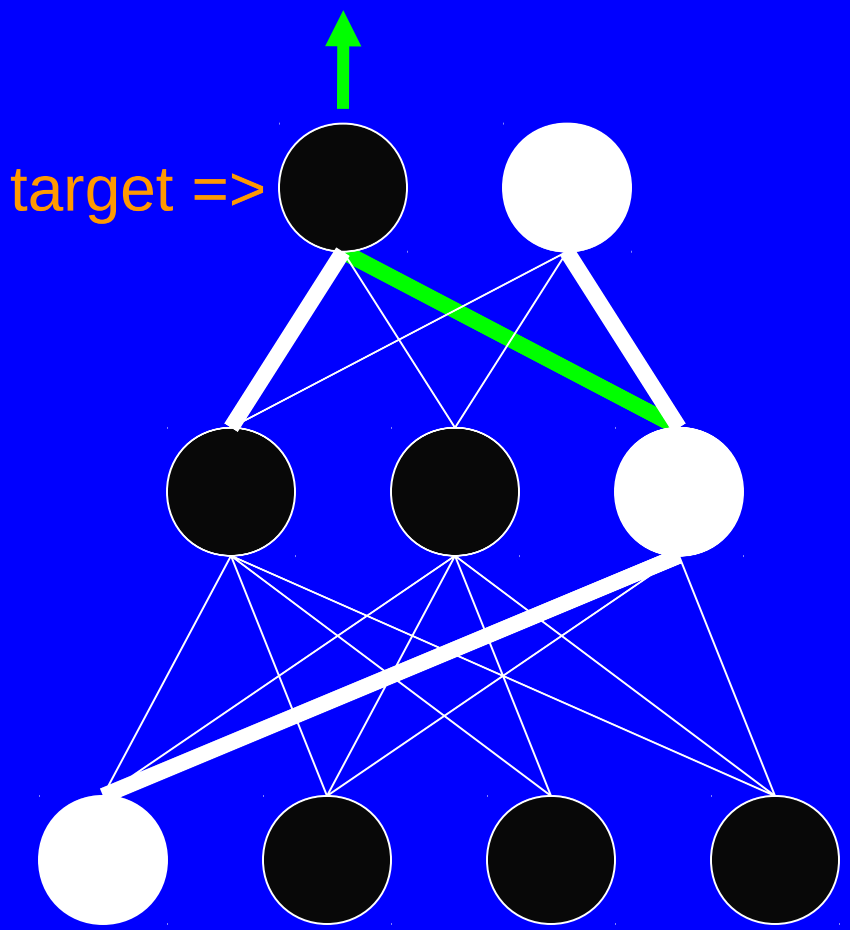

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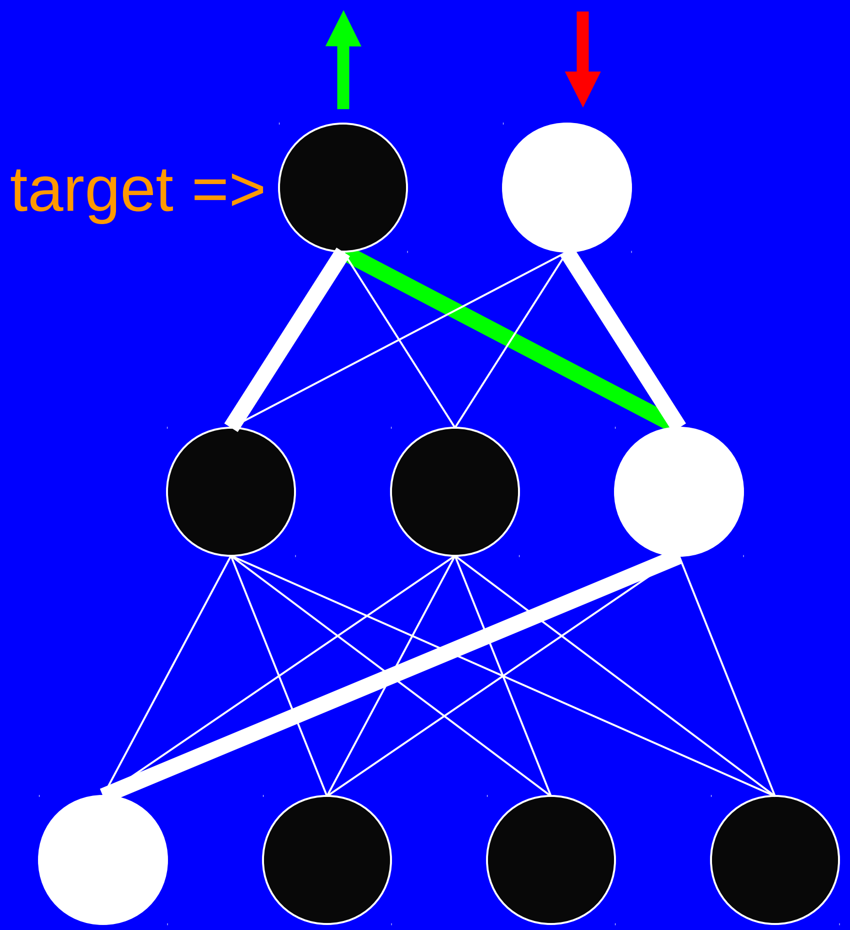

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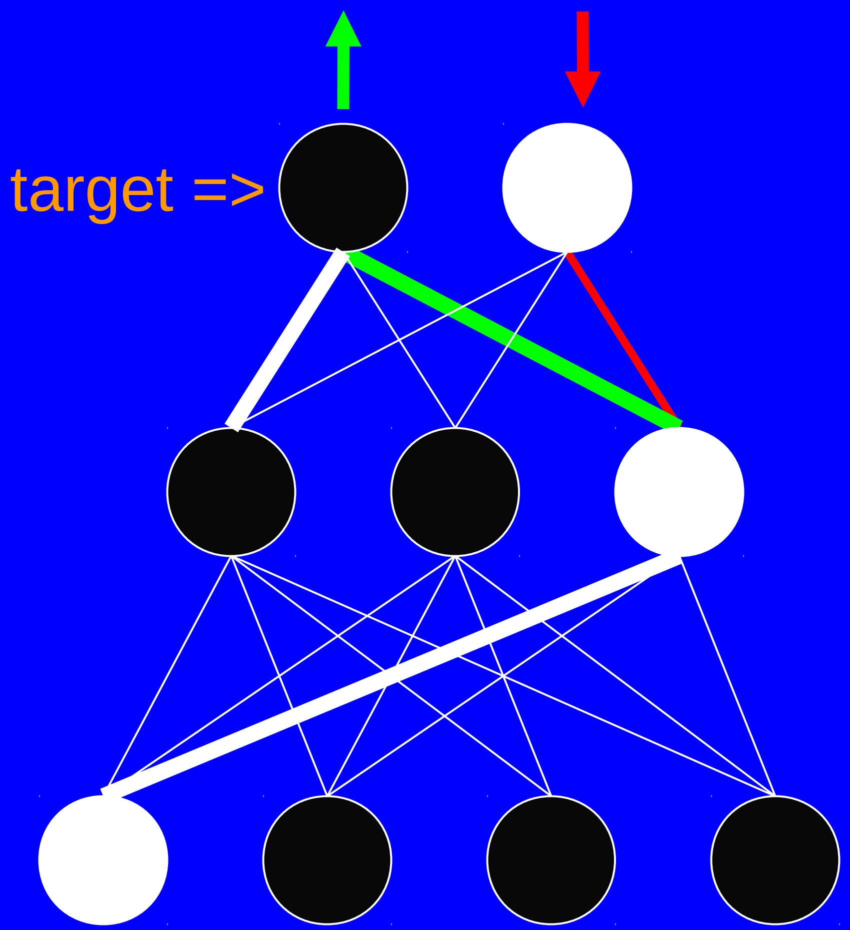

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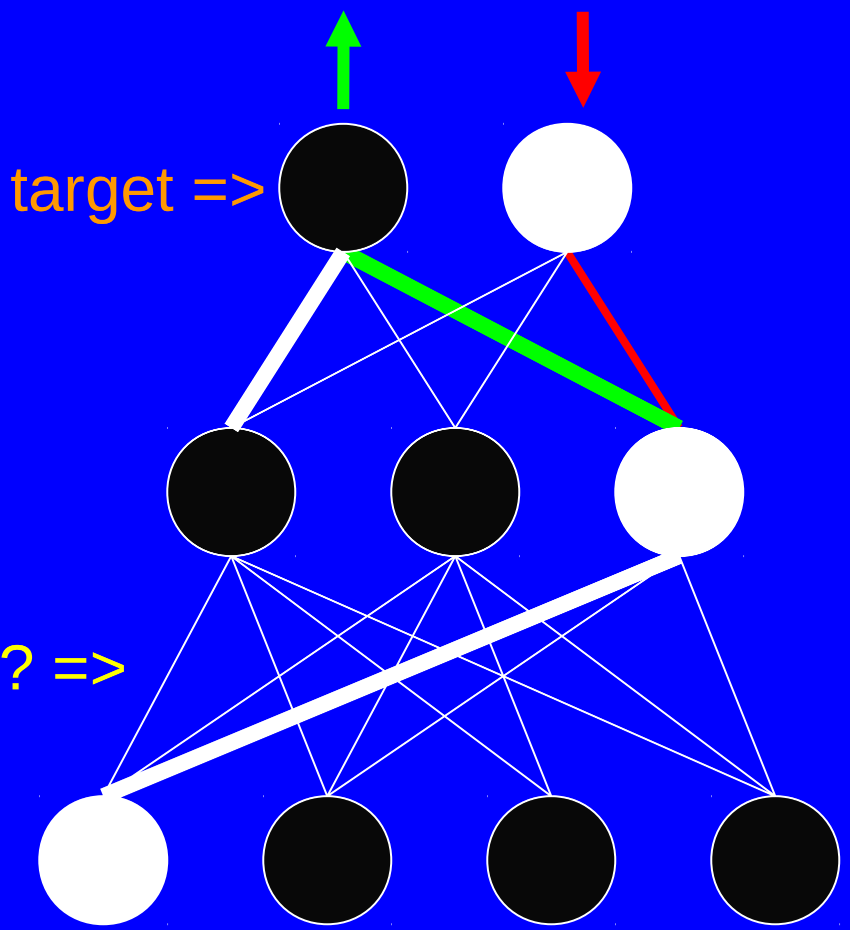
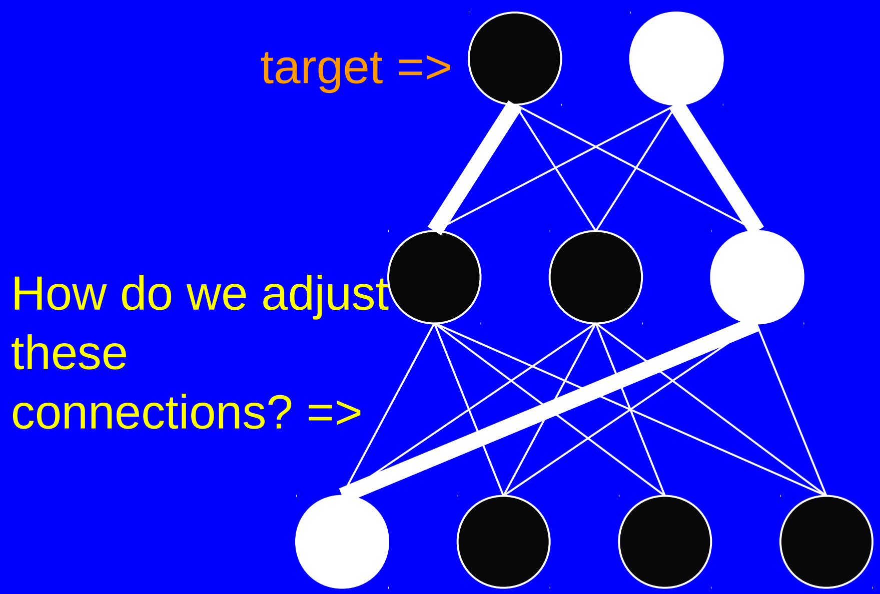


How do we
adjust these
connections? =>

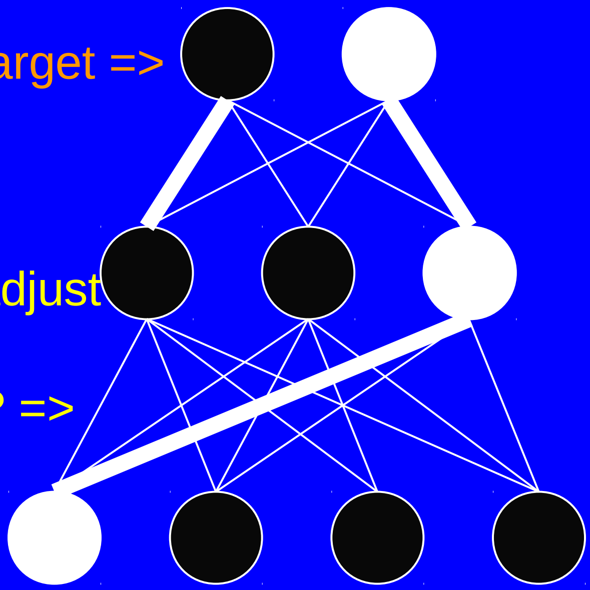
Learning in Multilayer Networks



Intuitively, you want to boost the activity of the middle guys that are well connected to the target unit

Learning in Multilayer Networks

target =>



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How do we identify units that are well connected to the target unit?

Learning in Multilayer Networks

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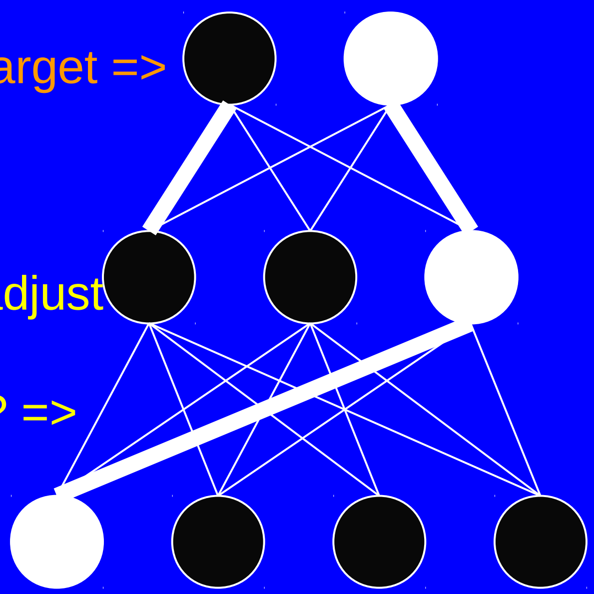

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How do we identify units that are well connected to the target unit?

Solution: Propagate activity backwards from the target

Learning in Multilayer Networks

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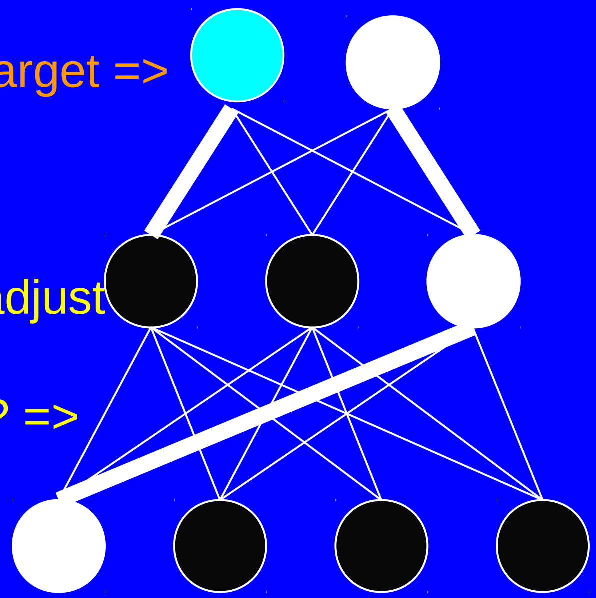

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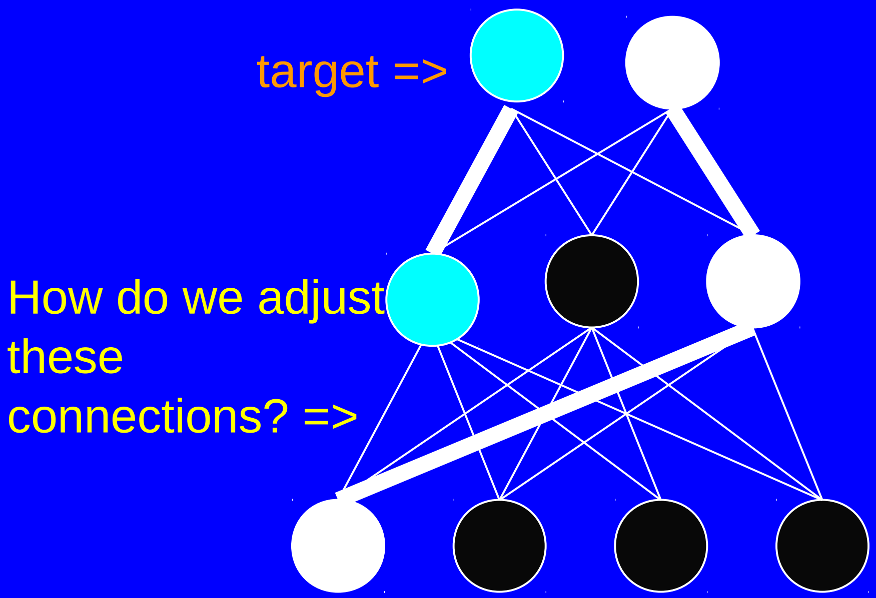

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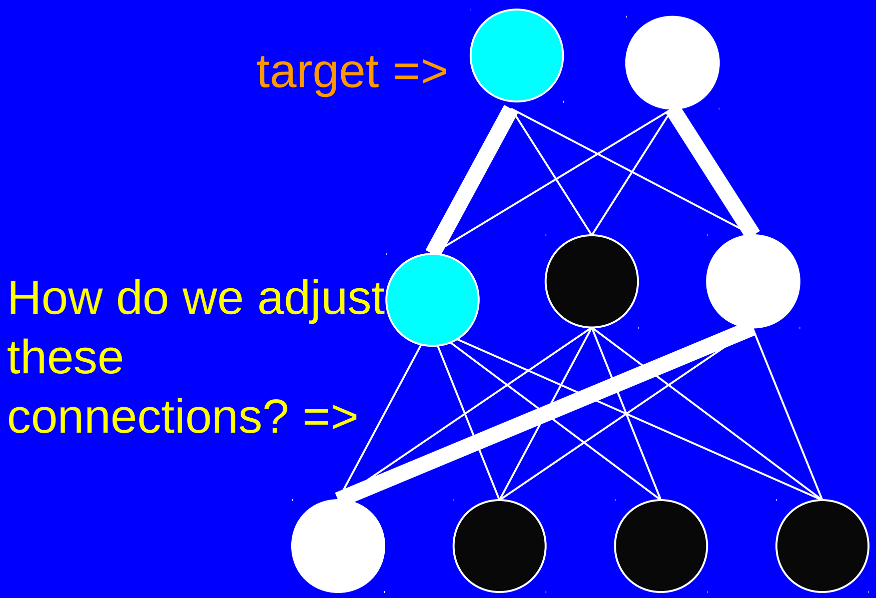

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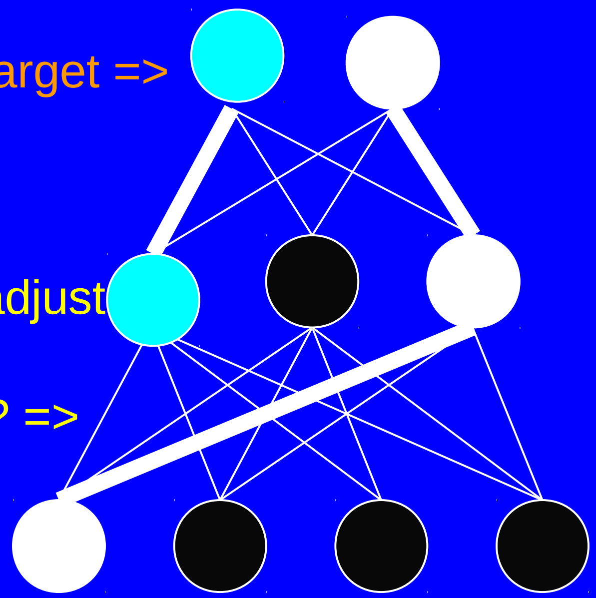
Learning in Multilayer Networks

target =>

How do we adjust these connections? =>



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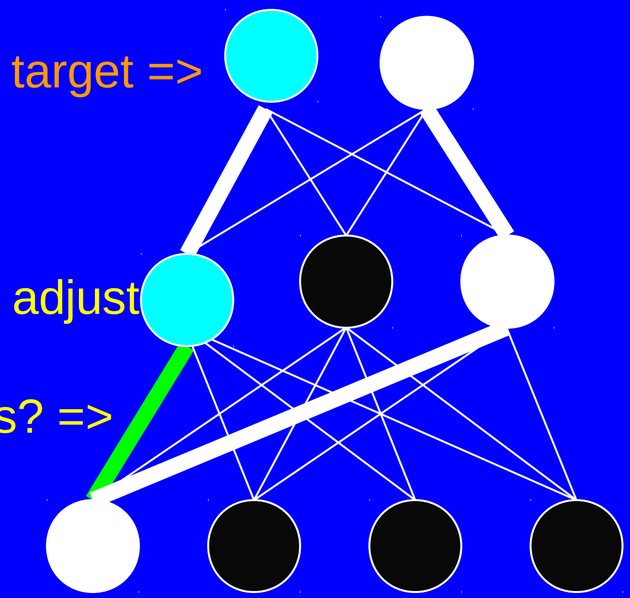
Then: change weights to strengthen these pathways

Learning in Multilayer Networks



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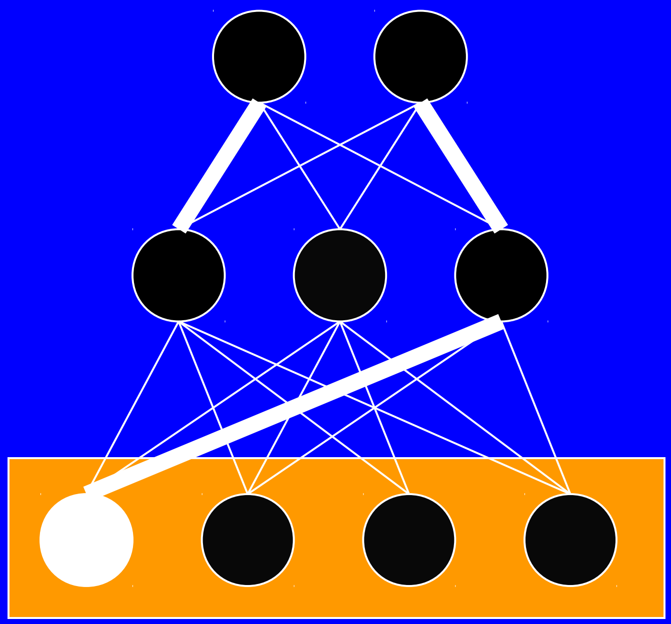
GeneRec Learning Rule

Compare two conditions:



Minus Phase:

Clamp input

GeneRec Learning Rule

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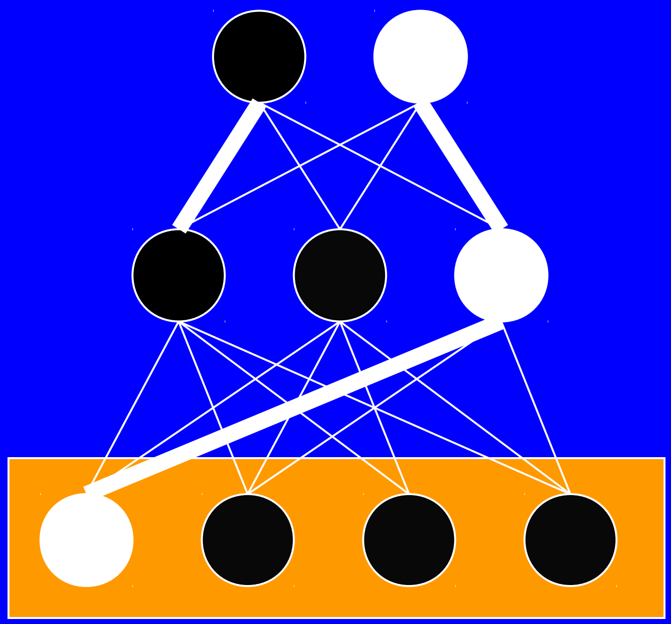

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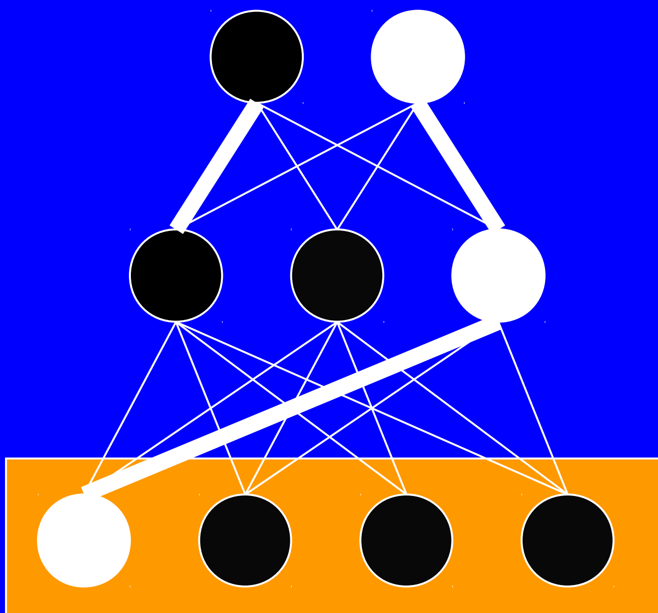

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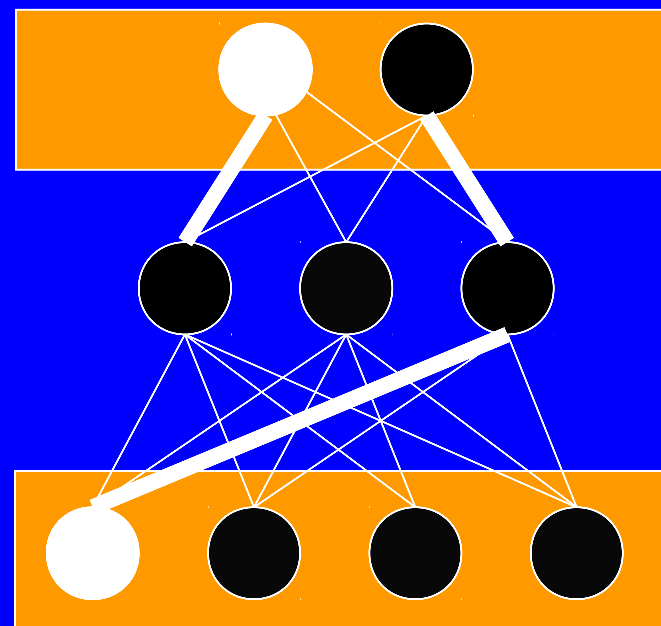

Plus Phase:
Clamp input and target output

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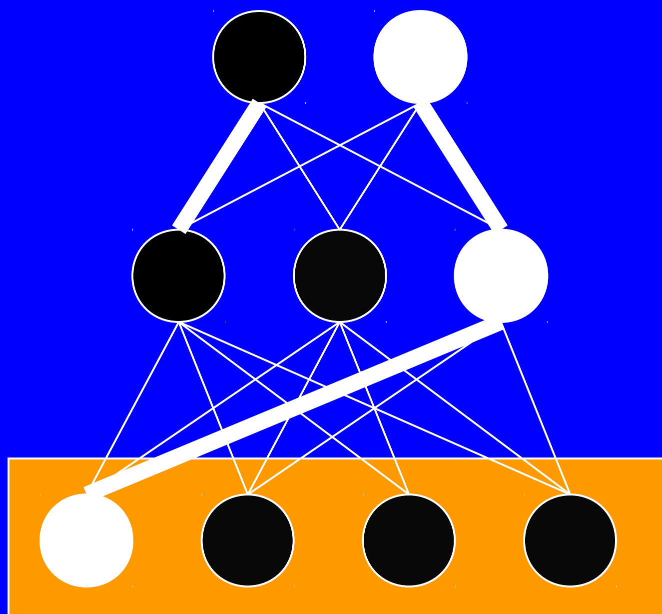

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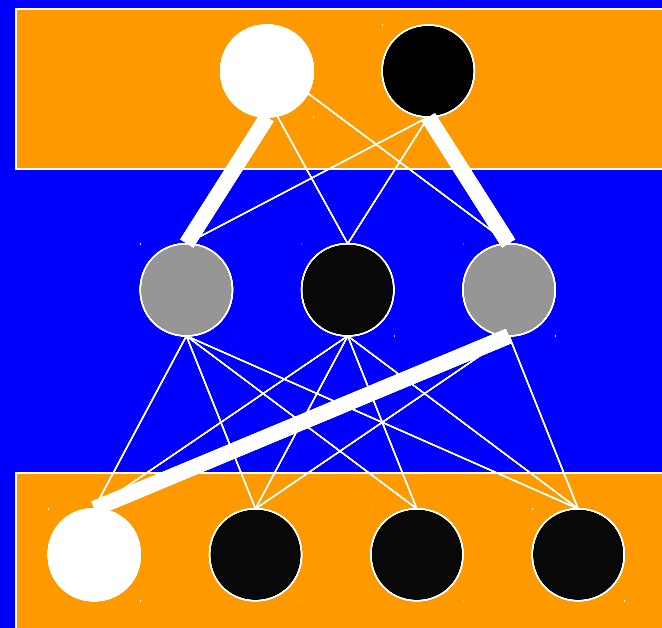

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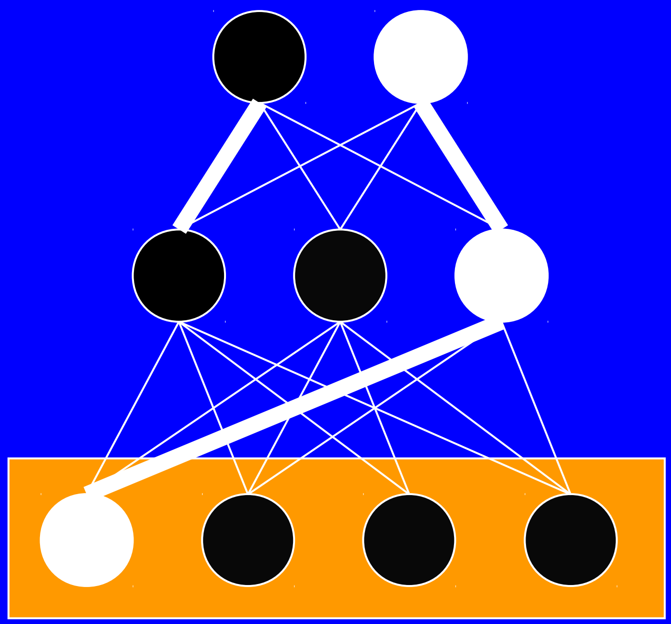

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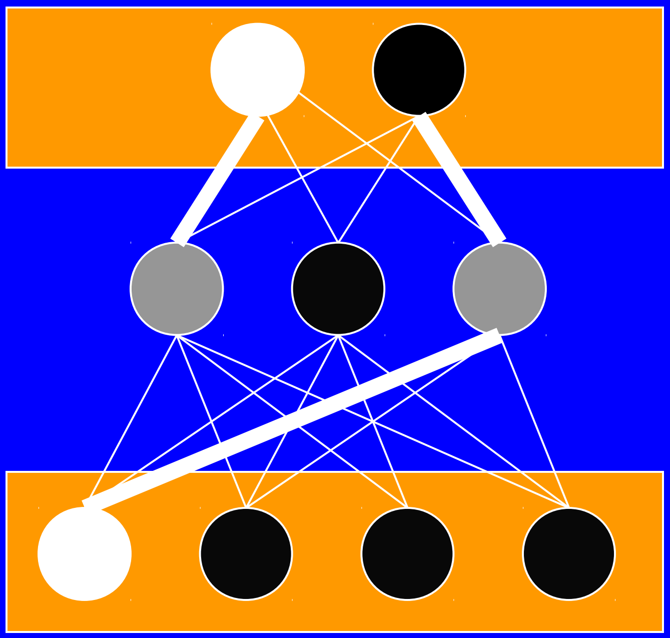

Plus Phase:
Clamp input and target output

For each layer, use the **difference between minus and plus activations** as an error signal and learn using the delta rule

GeneRec Learning Rule

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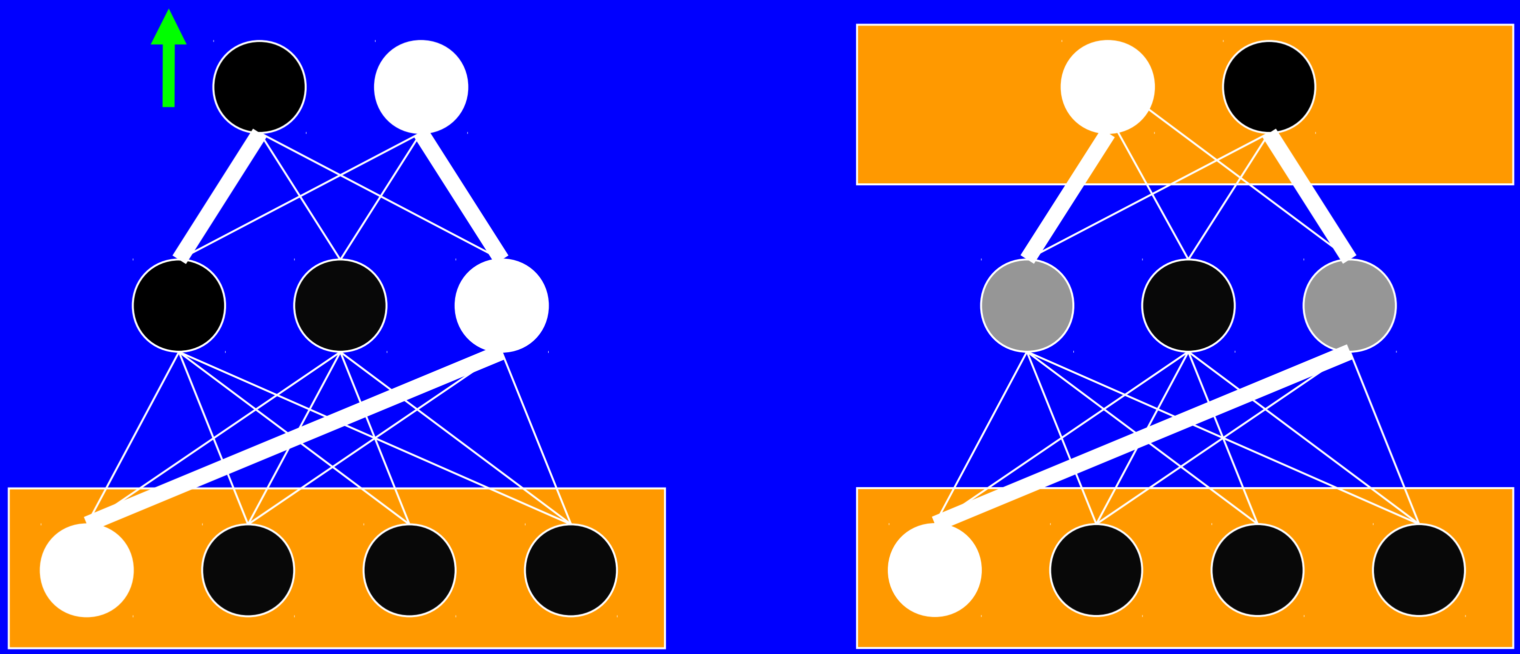

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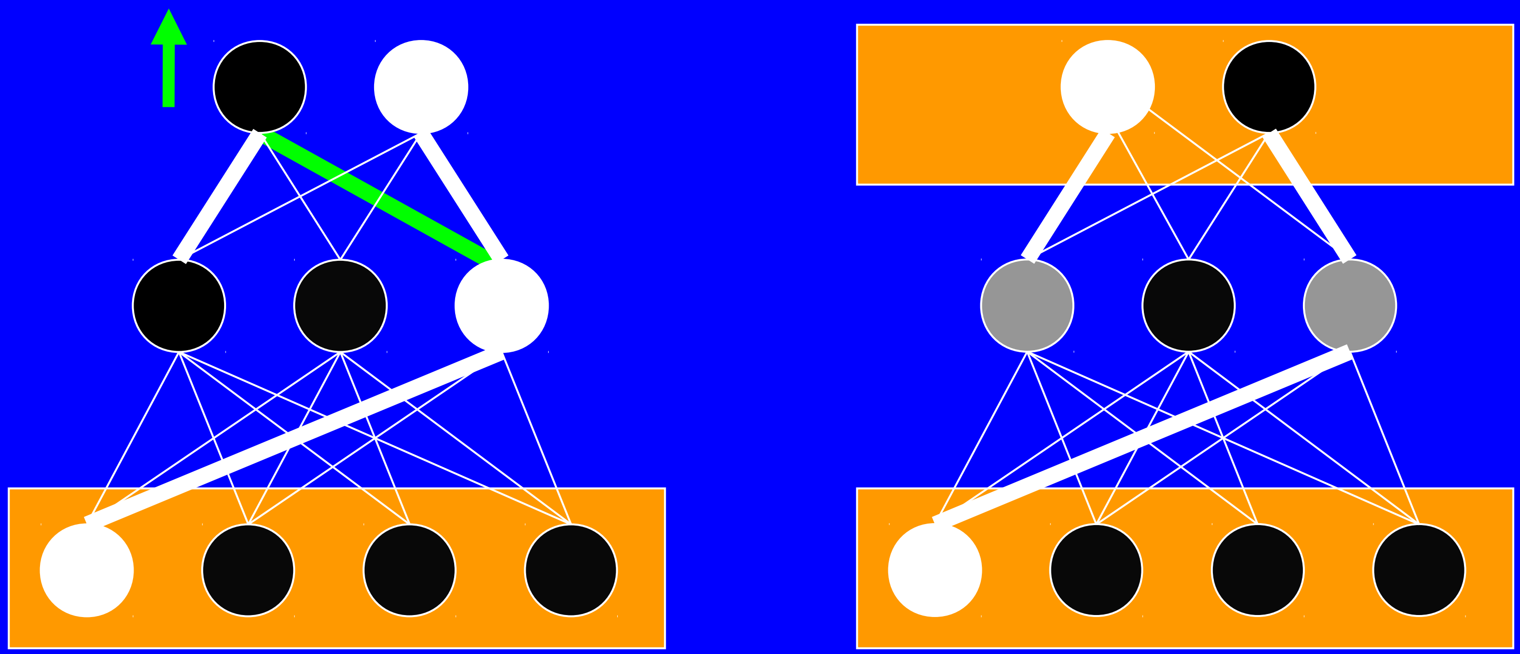

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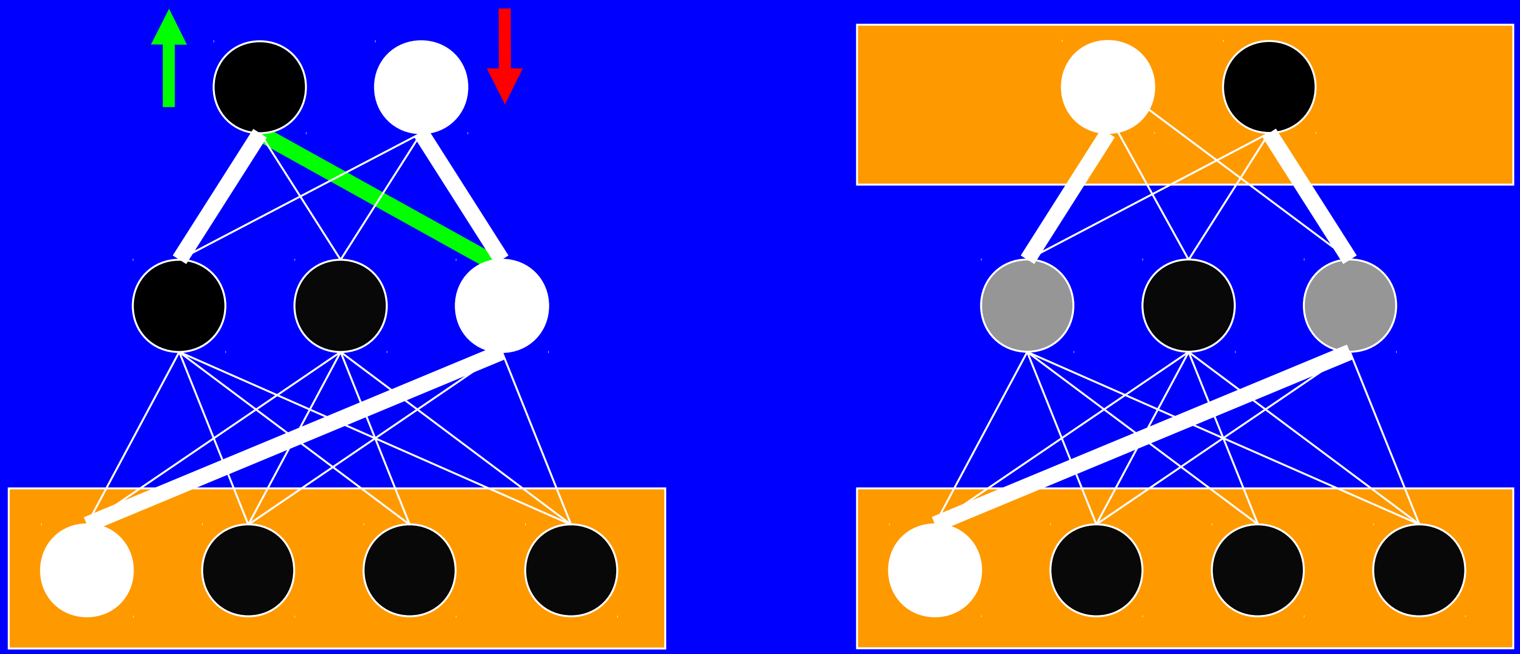

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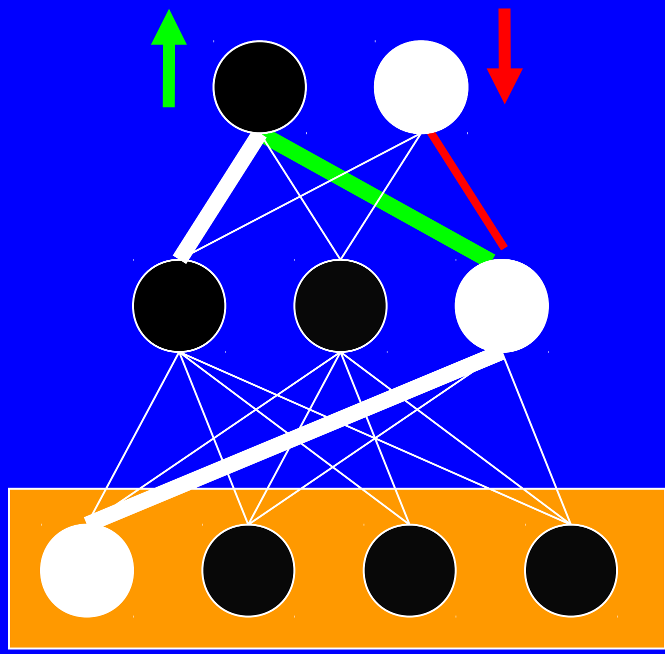

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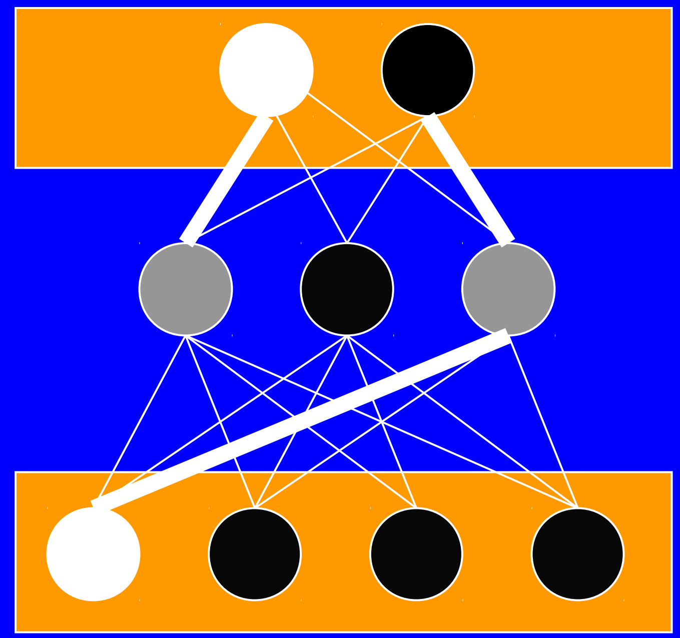

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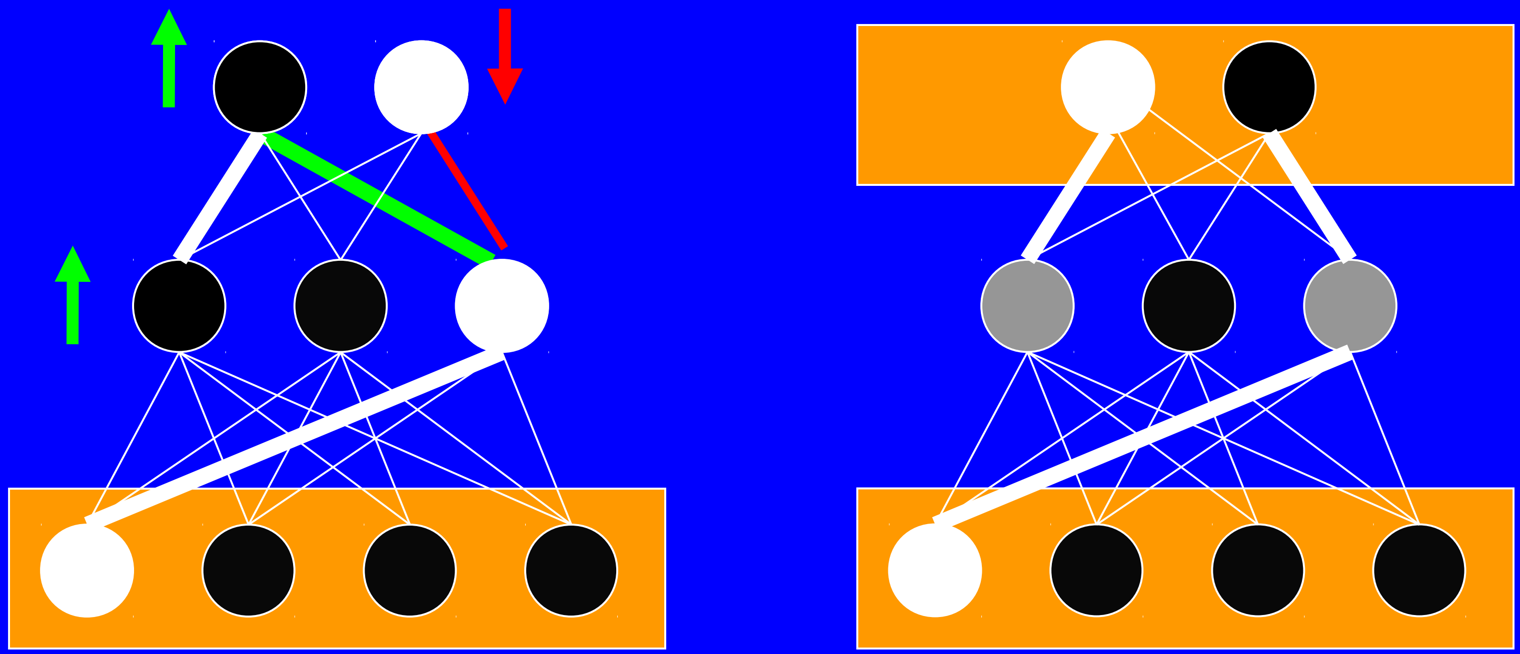

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GeneRec Learning Rule

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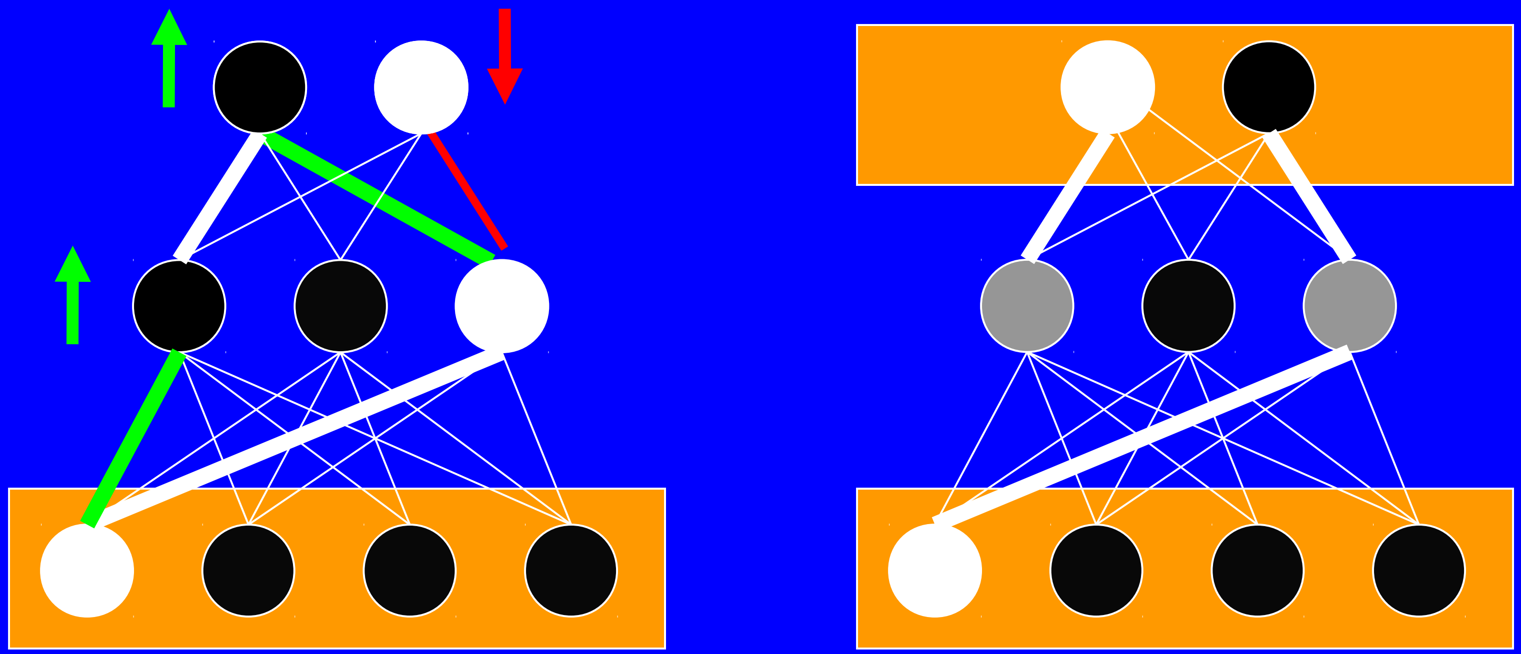

Minus Phase:
Clamp input

Plus Phase:
Clamp input and target output

For each layer, use the **difference between minus and plus activations** as an error signal and learn using the delta rule

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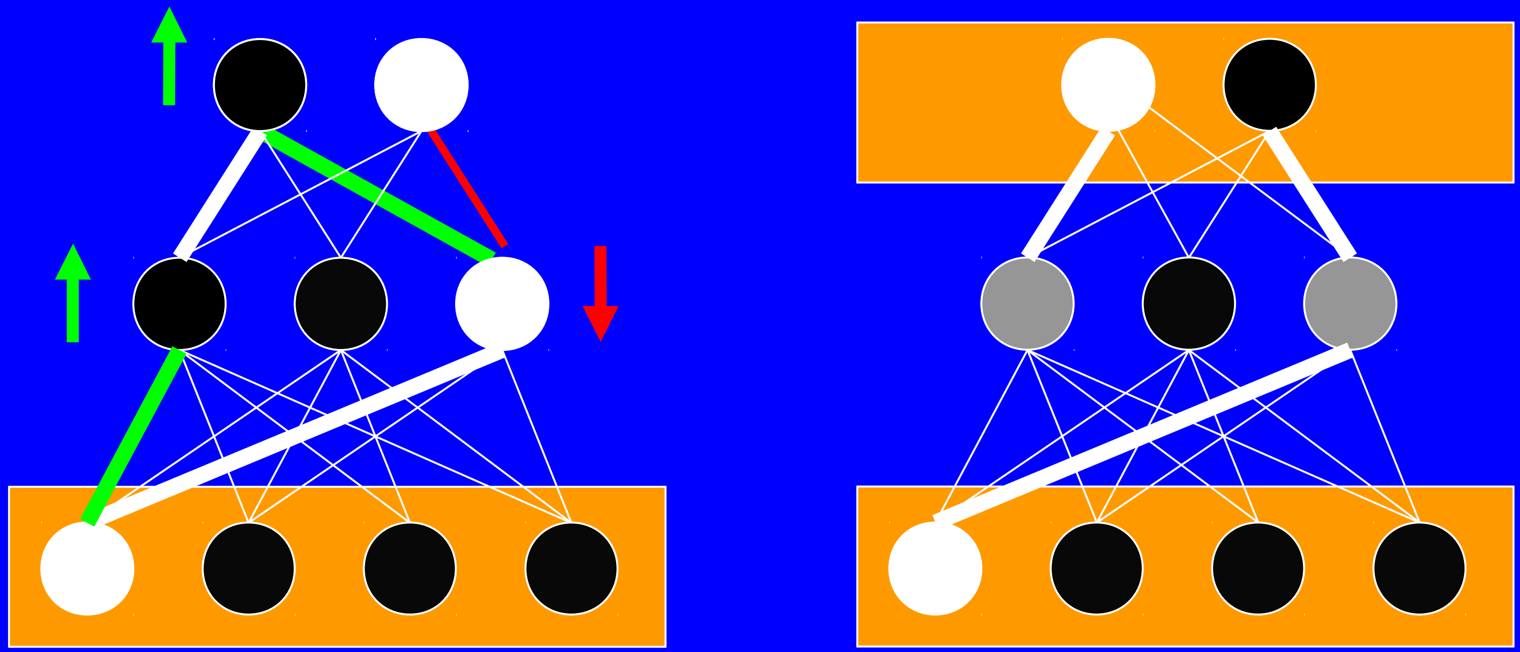

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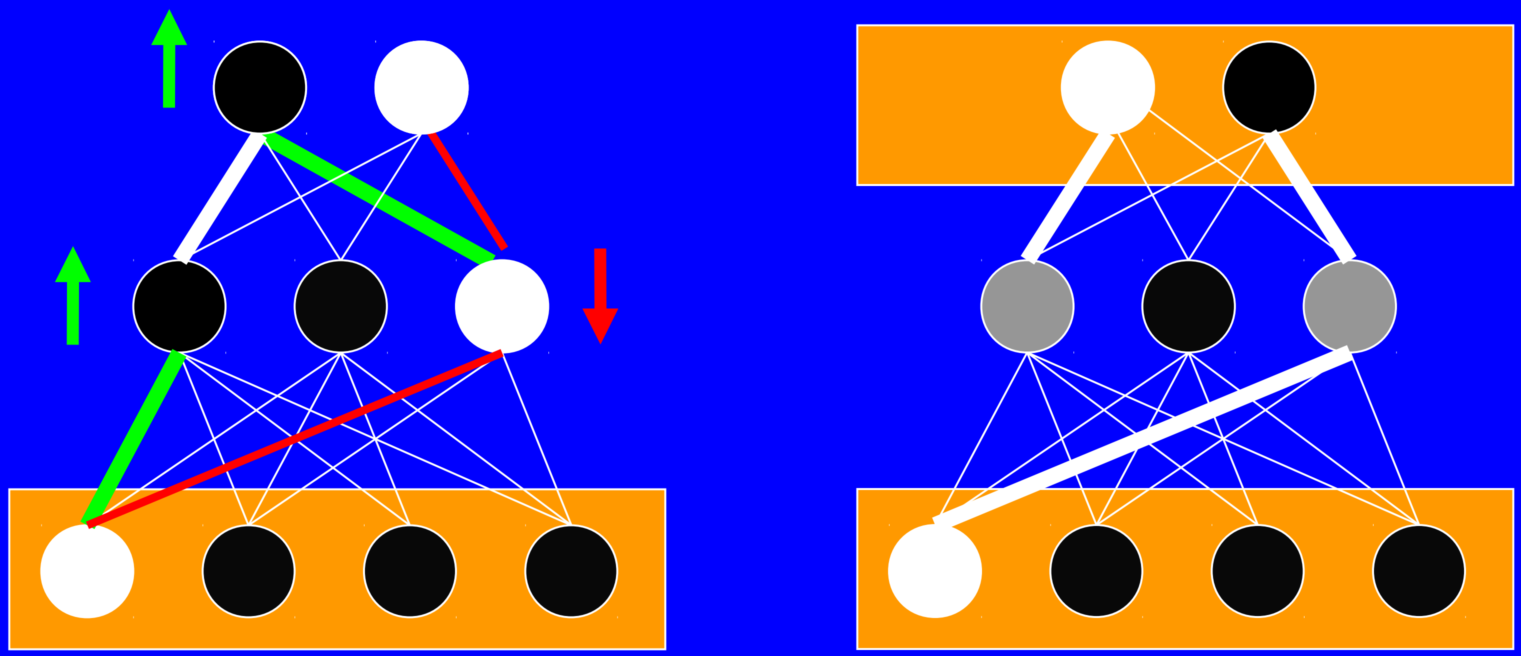

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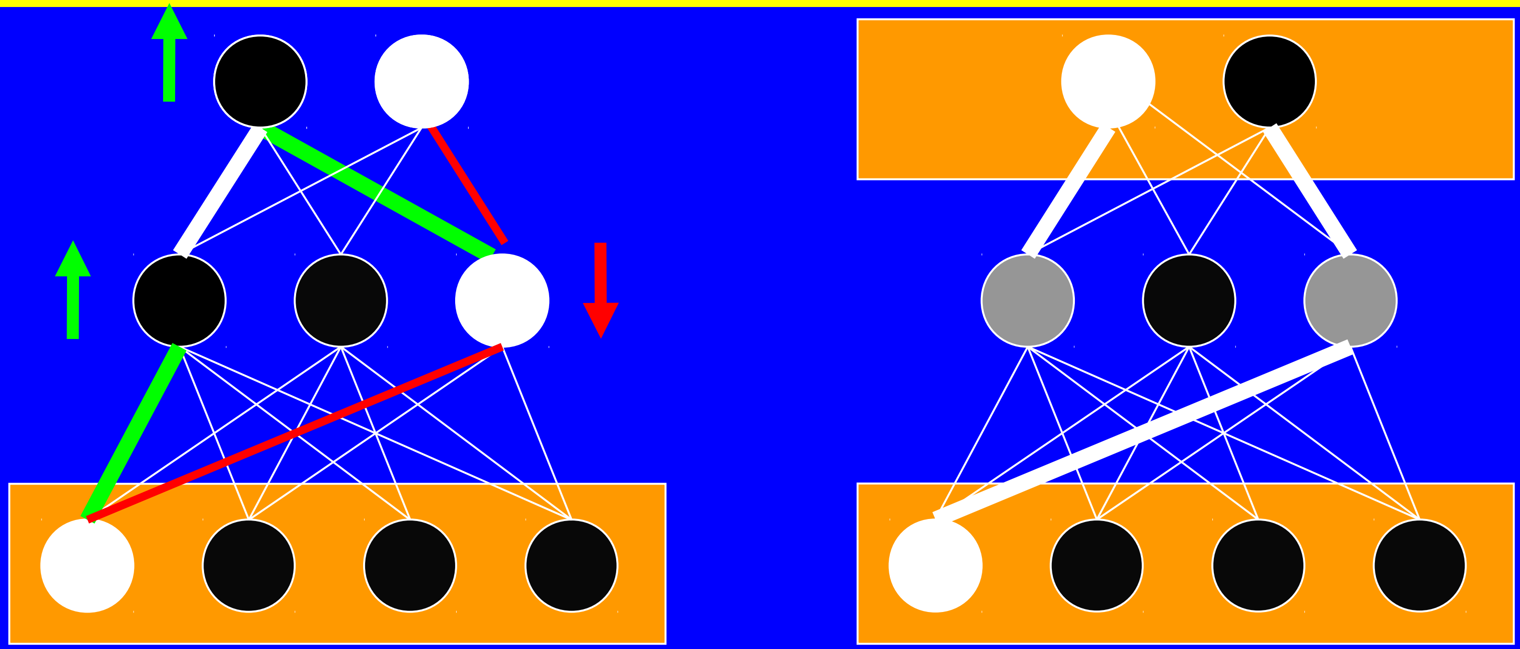
GeneRec: Summary

The goal of error-driven learning is to construct a **path** from the input to the target output



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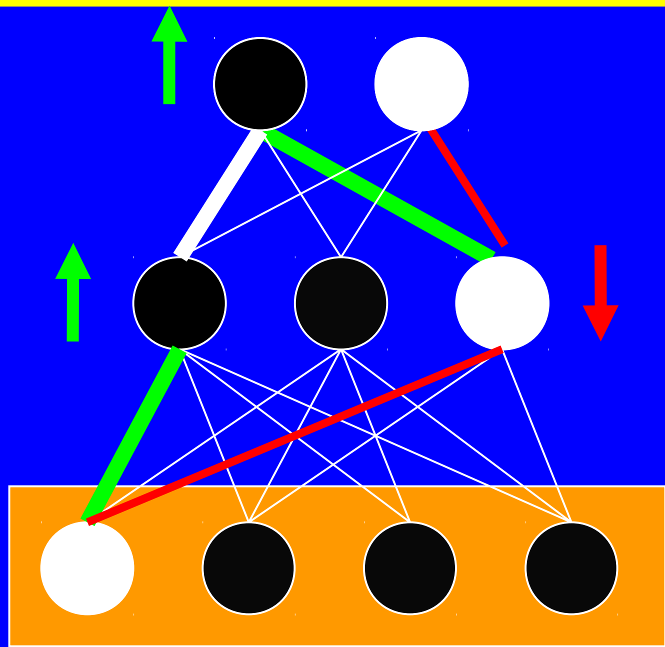

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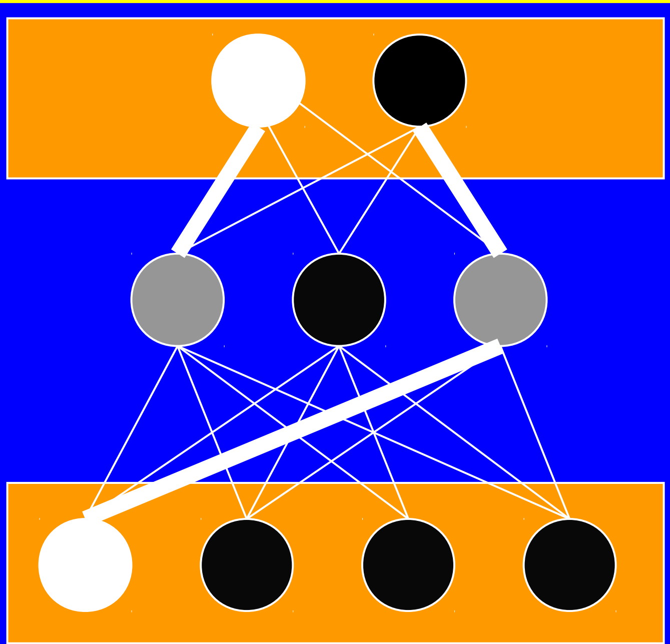
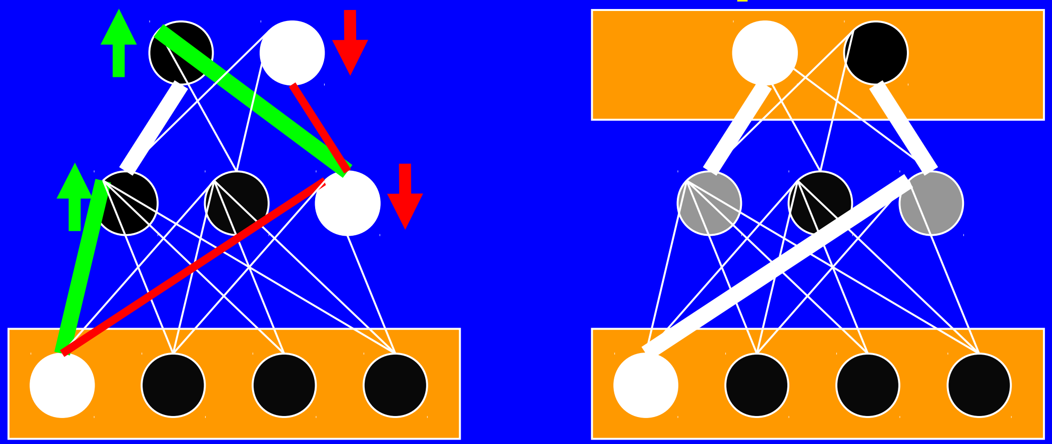


Plus Phase:

The Plus Phase helps identify **bridging units** that are well connected to both the input and the target output, and GeneRec adjusts weights to maximize the activity of these units

GeneRec: Equations



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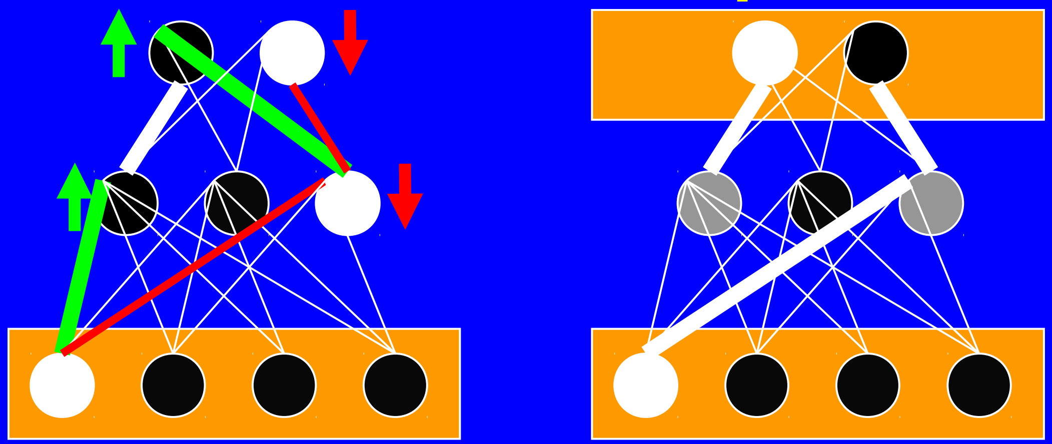
Basic GeneRec: $\Delta w_{ij} = \epsilon(y_j^+ - y_j^-)x_i^-$

GeneRec: Equations



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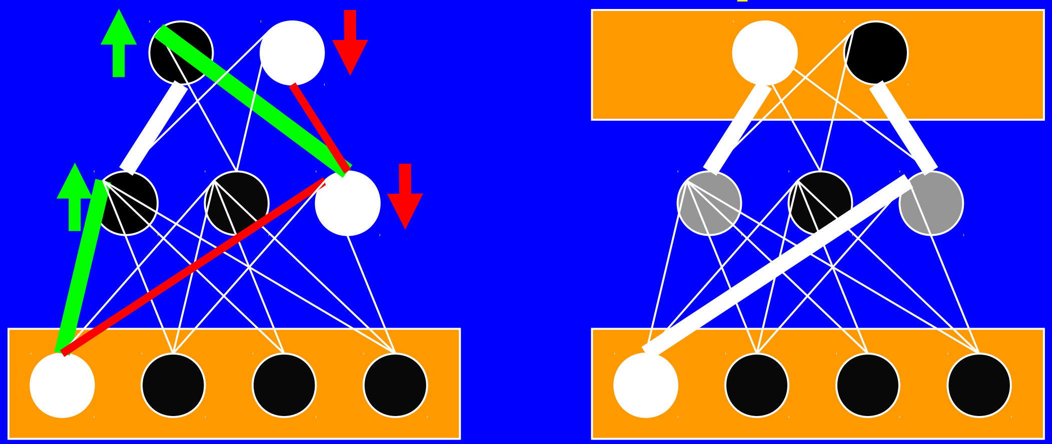

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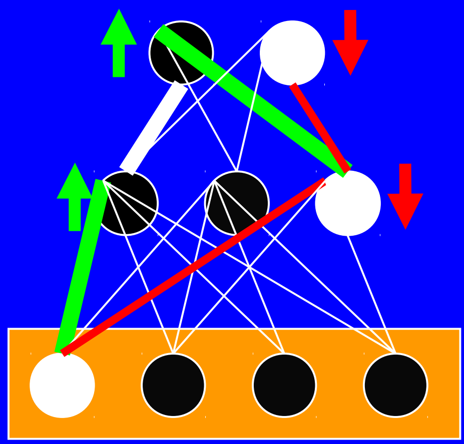
Solution: Average together plus and minus phase sending activation, and average together feedforward and feedback weight changes

GeneRec: Equations



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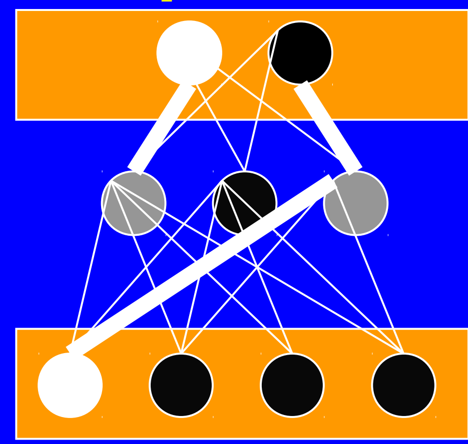
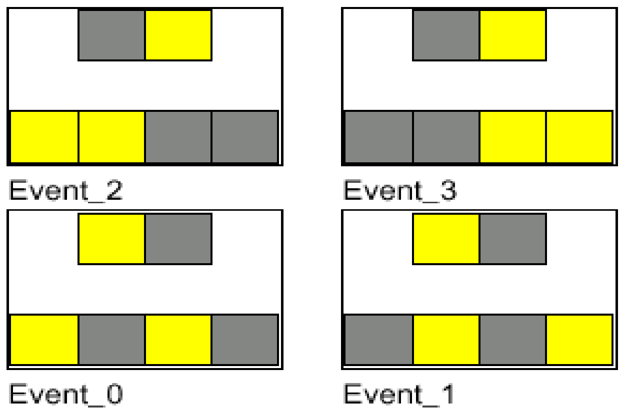


Plus Phase:
Clamp input and target output

Solution: Average together plus and minus phase sending activation, and average together feedforward and feedback weight changes

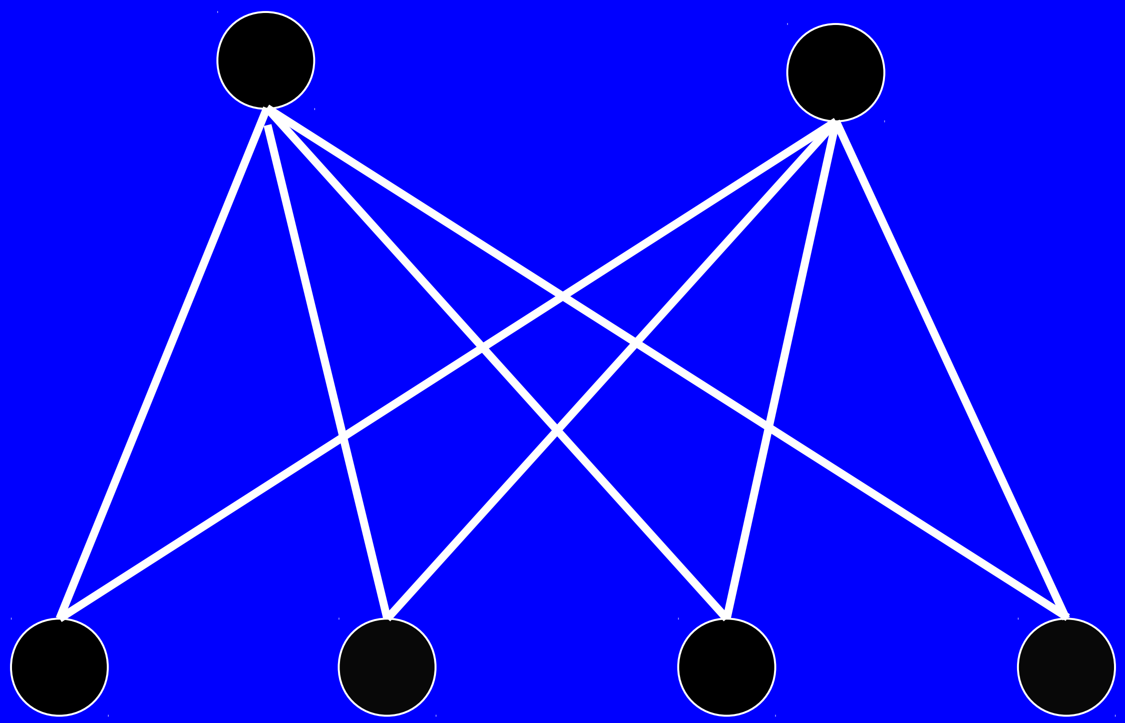
New and improved GeneRec: (CHL)

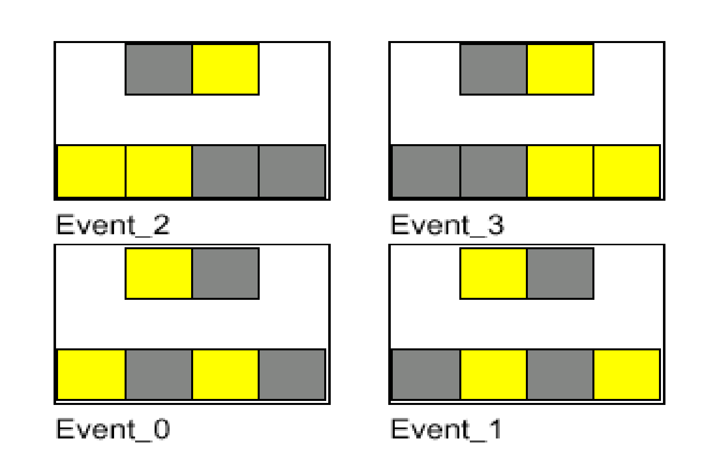
$$
\begin{aligned}
\Delta w_{ij} &= \epsilon \frac{1}{2}\left[(x_i^+ + x_i^-)(y_j^+ - y_j^-) + (y_j^+ + y_j^-)(x_i^+ - x_i^-)\right] \\
&= \epsilon \left[(x_i^+ y_j^+) - (x_i^- y_j^-)\right]
\end{aligned}
$$



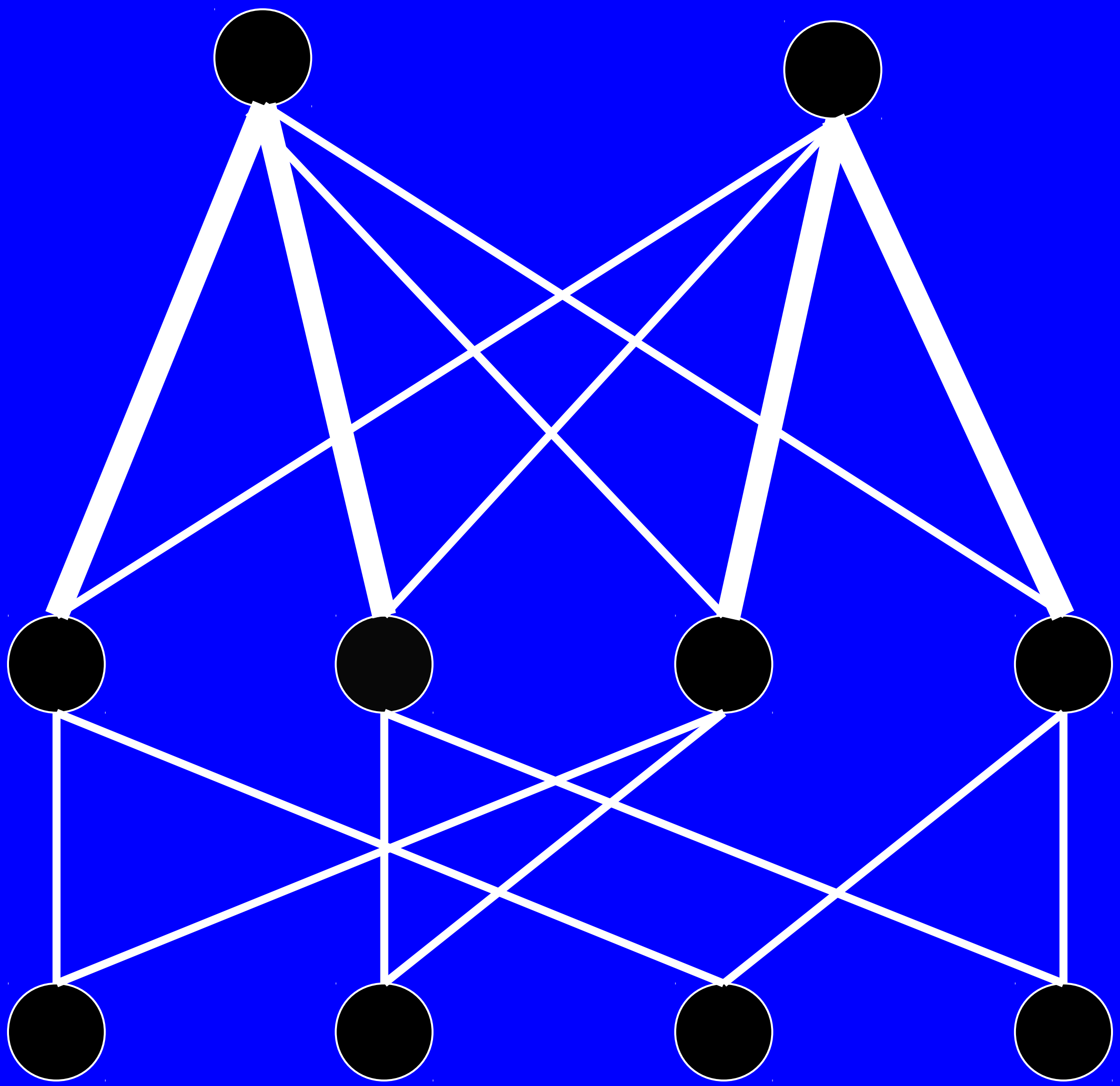
Remember the “impossible” problem?

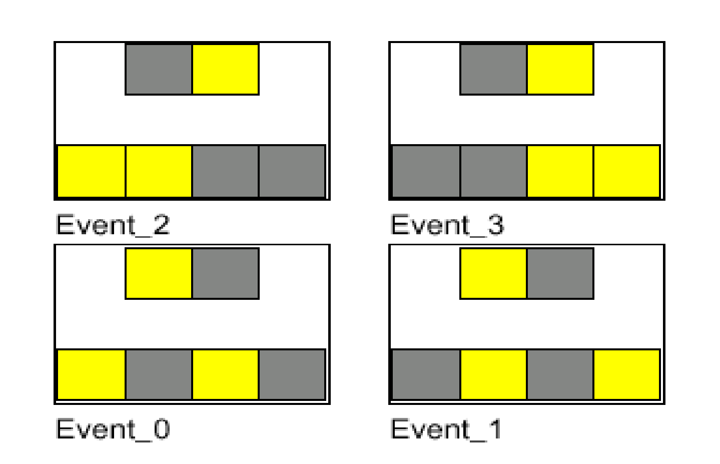
It can’t be solved by two-layer networks using the delta rule...





But it can be solved by three layer networks where hidden units represent feature conjunctions....





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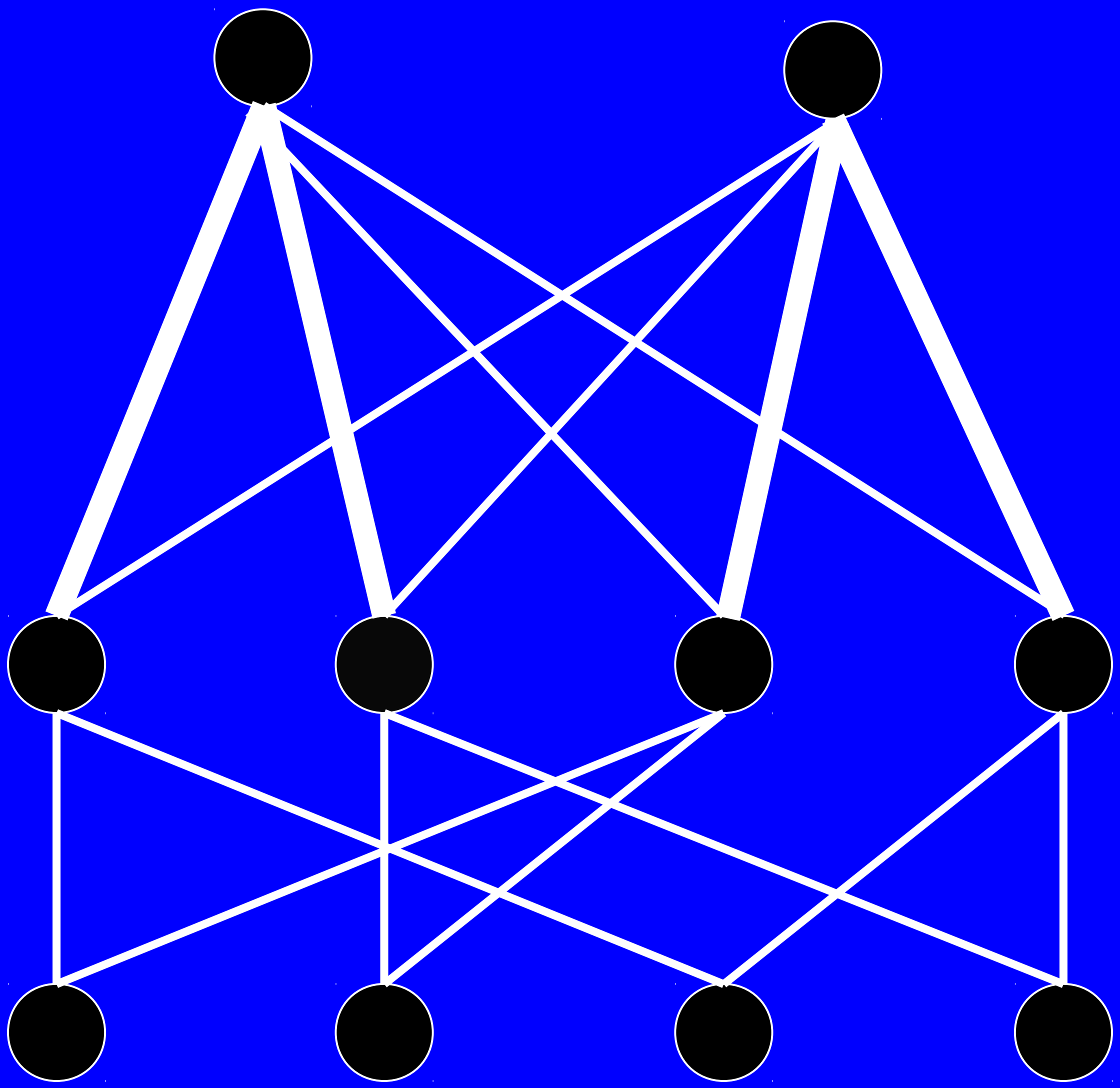
Does error-driven learning learn the correct set of weights?



Task Learning: Summary

- Hebbian learning alone is very limited in its ability to learn input-output mappings
  - If the input-output mapping happens not to coincide with the correlational structure of the inputs, Hebbian learning fails
- **Error-driven learning rules** (that leverage the difference between what the network was supposed to do, and what it actually did) do better at learning input-output mappings

Task Learning: Summary

- The **delta rule** can learn a wide variety of input-output mappings (including some that Hebb can not learn) in two-layer networks, but:
  - There are some mappings it can not learn (e.g., the “impossible” mapping)
  - It does not apply to networks with more than two layers

Task Learning: Summary

- The **GeneRec** rule remedies the deficiences of the simple delta rule
  - It applies to networks with hidden layers
  - It can solve tasks that can not be solved by the simple delta rule; this is accomplished by **re-representing** input patterns...
  - The rule is biologically plausible! Key prerequisites: Bidirectional connectivity, (approximate) symmetry, two “phases” (expectation and outcome)
- Next lecture: Synergies between Error and Hebb => Error + Hebb leads to better learning than Error alone!